Area accumulation functions and the FTC: an analytical perspective

1. Let $F(x) = \int_3^x e^{5t}\, dt$

   (a) Find a formula for $F(x)$ by anti-differentiating and substituting.

   (b) Differentiate to find $F'(x)$.

   (c) Explain your result.

   (d) Why does the lower limit of integration not affect the derivative?

   (e) Using what you noticed and learned above, find $\frac{d}{dx}\left[\int_{-5}^{x} \arctan t\, dt\right]$.

   In summary:

> The Fundamental theorem of Calculus, Part 1: If f is continuous on $[a, b]$, then
> $\frac{d}{dx}\left[\int_a^x f(t)\, dt\right] =$ ________ (for $a < x < b$).
> Worded differently, if $F(x) = \int_a^x f(t)\, dt$, then $F'(x) =$________.

2. Let $F(x) = \int_4^{x^2} \cos t\, dt$

   (a) Find a formula for $F(x)$ by anti-differentiating.

   (b) Differentiate to find $F'(x)$. Look at your answer and notice how it relates to the definition of $F(x)$.

(c) Using what you noticed and learned above, find $\dfrac{d}{dx}\left[\int_2^{\sin x} \ln t \, dt\right]$.

In summary:

If $F(x) = \int_0^{g(x)} f(t)\,dt$, then $F'(x) =$ ________________.

3. If $F(x) = \int_x^0 f(t)\,dt$, what is $F'(x)$? Hint: notice that $F(x) = -\int_0^x f(t)\,dt$.

4. If $F(x) = \int_{3x}^{x^2} \sin t\,dt$, what is $F'(x)$? Hint: the integral can be broken into two parts, so $F(x) = \int_{3x}^{0} \sin t\,dt + \int_0^{x^2} \sin t\,dt$.

In summary:

If $F(x) = \int_{a(x)}^{b(x)} f(t)\,dt$, then $F'(x) =$ ______________________________

5. Use the above result to answer the following: if $F(x) = \int_{x^3}^{1-x} \dfrac{t+1}{t-1}\,dt$, what is $F'(x)$?