MATH 1300: Diagnostic Exam

Name:

Directions: Circle the correct answer for each multiple choice problem below. This test should take you less than one hour.

1. \((x + 3)^2 =\)
   - (a) \(x^2 + 3\)
   - (b) \(x^2 + 9\)
   - (c) \(x^2 + 6x + 9\)
   - (d) \(x^2 + 9x + 9\)

2. \(\left(\frac{x^2 y^3}{x^2 y}\right)^6 =\)
   - (a) \(y^8 x^8\)
   - (b) \(y^3 x^{-8}\)
   - (c) \(4x^3 y^8\)
   - (d) \(x^4 y^{-2}\)

3. \(\frac{x}{x + 5} =\)
   - (a) \(\frac{1}{5}\)
   - (b) \(1 + \frac{1}{5}\)
   - (c) \(1\)
   - (d) None of the above.

4. \(\sin^2(\theta) + \cos^2(\theta) =\)
   - (a) 0
   - (b) 1
   - (c) \((\sin(\theta) + \cos(\theta))^2\)
   - (d) \(\tan^2(\theta)\)
5. \[ \frac{3}{4} + \frac{1}{3} - \frac{x}{6} = \]
   (a) \[ \frac{13 - 2x}{12} \]
   (b) \[ \frac{4 - x}{12} \]
   (c) \[ 4 - x \]
   (d) None of the above.

6. \[ 4^2 = \]
   (a) 6
   (b) 8
   (c) 64
   (d) None of the above.

7. If \( \sin(\theta) = \frac{1}{2} \) and \( \theta \) is in quadrant II, then \( \cos(\theta) = \)
   (a) \[ \frac{2\pi}{3} \]
   (b) \[ -\frac{1}{2} \]
   (c) \[ \frac{\sqrt{3}}{2} \]
   (d) \[ -\frac{\sqrt{3}}{2} \]

8. \[ \frac{x^{-2}}{y^2} = \]
   (a) \[ \frac{-x^2}{y^2} \]
   (b) \[ \frac{1}{x^2y^2} \]
   (c) \[ \frac{y^2}{x^2} \]
   (d) None of the above.

9. Simplify \( \frac{(x^2 + 2x - 3)(x + 2)}{(x + 2)(x - 1)} \).
   (a) \[ x + 3 \]
   (b) \[ \frac{x^3 + 4x^2 + x - 6}{x^2 + x - 2} \]
   (c) \[ \frac{x^2 + 2x - 3}{x - 1} \]
   (d) None of the above.
10. Simplify \( \frac{4x^2 + 6x}{2x} \).
   (a) 5
   (b) 5x
   (c) 2x + 3
   (d) None of the above.

11. \( \cos \frac{2\pi}{3} = \)
   (a) \( \frac{1}{2} \)
   (b) \( -\frac{1}{2} \)
   (c) \( \frac{\sqrt{3}}{2} \)
   (d) \( -\frac{\sqrt{3}}{2} \)

12. arctan \((-1)\) may equal which of the following?
   (a) 1
   (b) \( \frac{\pi}{4} \)
   (c) \( -\frac{\pi}{4} \)
   (d) \( \frac{\sqrt{2}}{2} \)

13. Simplify the following expression: \( \sqrt{49 + \pi^2} \)
   (a) 49 + \( \pi \)
   (b) 7 + \( \pi \)
   (c) 7 - \( \pi \)
   (d) The expression cannot be simplified further.

14. Simplify the expression: \( \sqrt{(2x^2 \sqrt{y})^4} \)
   (a) \( \frac{4x}{\sqrt{y}} \)
   (b) \( 4x^2 y^4 \)
   (c) \( 16xy^2 \)
   (d) \( 4x^4 y \)
   (e) \( \sqrt[4]{(2x^2 \sqrt{y})} \)

15. Simplify the expression: \( \frac{\cos x}{\cos x \sin^2 x + \cos^3 x} \)
   (a) \( \cos x \)
   (b) 1
   (c) \( \frac{1}{\cos x} \)
   (d) \( \sin x \)
16. Solve the equation \( e^{4x-1} = 1 \).

(a) \( \frac{1}{2} \)  
(b) 0  
(c) \( \frac{1}{4} \)  
(d) \( \ln \frac{1}{4} \)  
(e) no solutions

17. Evaluate \( \log_2 \frac{1}{16} \)

(a) 4  
(b) 8  
(c) -4  
(d) -8  
(e) 2

**Short Answer**

18. Find the equation for a circle with radius 3 and center \((-1, 2)\)

19. Find the vertex of the parabola \( y = 2x^2 + 3x - 5 \)
20. If an object makes 3 rotations per minute around a circle of radius 3 ft, determine its angular and linear velocities.

21. Two cars start at the same point. Car A heads due south at 60 km/h, while car B heads due east at 80 km/h. How far apart are the two cars after 2 hours?

22. A circular cone with a base radius of 12 cm and a height of 4 cm is turned upside down (standing on its vertex) and filled with water. What is the total volume of water when it has a depth of 6 cm? (Remember that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.)

23. A man is standing 8 ft from a light pole. At that point he is casting a 12 ft shadow. If the man is 6 ft tall, how high is the light pole?
24. What is the $y$-intercept of the function $x^3 - 4x^2 + 12x - 25$

25. Find the solution to: $\left| \frac{5-3x}{4} \right| < 5$.

26. Find the distance between the two points $(-1, 2)$ and $(0, 4)$.

27. Find the slope of the line passing through the points $(-5, -2)$ and $(1, 4)$. 
28. Factor: \(2x^3 + 8x^2 - 3x - 12\)

29. Solve the equation \(\log_3 (x + 6) - \log_3 (x - 2) = 2\)

30. The population of a certain species of bacteria is given by \(P(t) = 500(1.3)^t\). How long will it take the population to double? (Do not use a calculator for this problem. Your answer should be in terms of logarithms and numbers)

31. Find an equation for a polynomial with zeros at \(x = -2, x = 1,\) and \(x = 3\).