

## The 29<sup>th</sup> Annual Great Plains Operator Theory Symposium

Titles and abstracts of plenary talks (all plenary talks held in HUM 150)

### Tuesday June 2, 9:30 a.m.: George Elliott, University of Toronto

The classification question for non-simple inductive limits of matrix algebras over one-dimensional spaces

**Abstract:** Simple inductive limits of (sequences of) matrix algebras over one-dimensional (metrizable) locally compact spaces were classified by Liangqing Li, after earlier work by me in the case of circles. It turns out that, as a consequence of the work of Li, the case of circles exhausts the simple inductive limits. The invariants for this class of algebras, say assumed to be stable, may be viewed as just the Cuntz semigroup together with the Banach algebra  $K_1$ -group.

In the non-simple case, if the Banach algebra  $K_1$ -group is assumed to be zero, then it was shown recently by Ciuperca, Robert, Santiago, and me that the Cuntz semigroup is a complete invariant. (Again, in the stable case—otherwise one needs to keep track of the canonical Cuntz class—which is just the largest element of the semigroup in the stable case.) (Ciuperca and I had obtained this result in the case of closed or half-open intervals in the line, building on work of Coward, Ivanescu, and me on the general theory of the Cuntz semigroup, and in particular on the fact that, in a suitable sense, this invariant is preserved under passage to inductive limits—and building also on work of Robert, who obtained essentially the same result (closed or half-open intervals) in an important case (totally ordered ideal lattice for the limit algebra) but without mentioning the Cuntz semigroup; it only became clear in fact after the work of Ciuperca and me, in work of Robert, Santiago, and me, that Robert’s invariant, in the case that he considered, actually was the Cuntz semigroup. How to calculate the Cuntz semigroup in more general cases, even in the case that the spaces are closed or half-open intervals, is still an interesting problem.)

The general case, in which one has to consider not only the Banach algebra  $K_1$ -group, but also the algebraic  $K_1$ -group, and not only for the whole algebra, but also inside each hereditary sub- $C^*$ -algebra—equivalently, in the present case (in which the stable rank is one), inside each Cuntz element—seems not quite to be within reach at the moment. The case that the algebra has the so-called ideal property—ideals generated by projections—, dealt with by K. Stevens in the setting of closed intervals, appears to be manageable (work in progress with Ciuperca, Robert, and Santiago).

### Tuesday June 2, 11:00 a.m.: Iain Raeburn, University of Wollongong

Phase transitions in a number-theoretic dynamical system

**Abstract:** KMS states are the manifestation of equilibrium states in  $C^*$ -algebraic models of statistical-mechanical systems. Over the past 15 years, there have been many intriguing examples of systems arising in number theory whose KMS states exhibit the sort of properties one expects of physical models.

In this talk we will consider dynamical systems consisting of an action of the real line on a  $C^*$ -algebra. We will explain what KMS states of such a system are, and illustrate the general ideas with a discussion of what happens for the gauge action on the Cuntz algebra and

its Toeplitz analogue. We will then discuss current joint work with Marcelo Laca in which we determine the KMS states on the Toeplitz algebra of a number-theoretic version of the  $ax + b$  group, and find some intriguing new phenomena.

**Tuesday June 2, 1:30 p.m.: Guihua Gong, University of Puerto Rico**

Title: TBA

**Wednesday June 3, 9:00 a.m.: Kathy Merrill, Colorado College**

Equivalence parameters and a canonical construction for GMRAs

**Abstract:** (Joint work with L. Baggett, V. Furst and J. Packer.) A generalized multiresolution analysis is a Hilbert space structure closely associated with wavelets. GMRAs first appeared as a way to describe wavelets in  $L^2(\mathbb{R}^n)$ , such as the Journé wavelet, that are not associated with a classical multiresolution analysis. More recently, GMRAs have been used to develop wavelets in more general Hilbert spaces, such as those built from functions on fractals or by direct limit constructions. In this talk, we discuss a notion of equivalence among GMRAs, and describe an explicit construction of a canonical GMRA for each equivalence class.

**Wednesday June 3, 10:00 a.m.: Raul Curto, University of Iowa**

Cubic Column Relations in Truncated Moment Problems

**Abstract:** In joint work with Seonguk Yoo we consider truncated moment problems and cubic column relations of the form  $Z^3 = itZ + u \text{ conj}(Z)$ , where  $u$  and  $t$  are real numbers. We prove that the algebraic variety consists of exactly 7 points, and we then apply the solution of the extremal moment problem (found in joint work with Fialkow and Moller) to obtain NASC for the existence of a representing measure when these cubic relations arise.

**Wednesday June 3, 11:00 a.m.: Paul Baum, Pennsylvania State University**

What is an equivariant index ?

**Abstract:** Let  $G$  be a (countable) discrete group acting by diffeomorphisms on a smooth manifold  $M$ . Assume that the action is smooth, proper, and co-compact. Let  $D$  be a  $G$ -invariant elliptic differential (or perhaps pseudo-differential) operator on  $M$ . What should we mean by the equivariant index of  $D$ ? This talk will take up this issue and will explain how equivariant index is used in the BC (Baum-Connes) conjecture. An example is  $G = \mathbb{Z}$ ,  $M = \mathbb{R}$ , the real line, and  $D = -i\frac{d}{dx}$ .  $\mathbb{Z}$  acts on  $\mathbb{R}$  by the usual translation action. This example will be considered in detail.

This talk is intended for non-specialists. The basic definitions will carefully stated.

**Wednesday June 3, 1:30 p.m.: Marcelo Laca, University of Victoria**

On the Toeplitz algebra of the affine semigroup over the natural numbers.

**Abstract:** We give a presentation of the Toeplitz algebra of the semidirect product semigroup of the additive by the multiplicative natural numbers, and conclude that the  $C^*$ -algebra  $Q_N$  recently introduced by Cuntz is a (boundary) quotient of this Toeplitz algebra. We then com-

pute the KMS states of the natural Toeplitz dynamical system; it has a phase transition with spontaneous symmetry-breaking and we analyze it in relation to various symmetries of the system. This is joint work with Iain Raeburn.

**Thursday June 4, 9:00 a.m.: Dietmar Bisch, Vanderbilt University**

Bimodules, planarity and freeness

**Abstract:** Bimodules can be viewed as natural symmetries of noncommutative measure spaces. They form fusion categories whose intertwiner spaces carry an action of Jones' planar operad. This gives rise to many surprising algebraic-combinatorial structures, including a novel notion of "free product" for planar algebras. This idea leads to new infinite depth subfactors and new structure theorems for subfactors. The talk is based on joint work with Vaughan Jones.

**Thursday June 4, 10:00 a.m.: Magnus Landstad, NTNU, Norway**

Can a function and its Fourier transform both have compact support? And other questions with the same answer

**Abstract:** It is well known that there are no non-zero continuous functions on  $\mathbb{R}$  with the property that both the function and its Fourier transform have compact support. If we replace  $\mathbb{R}$  with any locally compact abelian group, is the same thing true?

We shall see that the answer to this question is the same as to the following: It is folklore that the product of a multiplication operator and a convolution operator is compact. When is this product a finite rank operator? When does a multiplication operator and a convolution operator commute?

Only basic knowledge about groups, Fourier transforms and Hilbert spaces is assumed.

This is joint work with A. Van Daele.

**Thursday, June 4, 11:00 a.m.: Christopher Seaton, Rhodes College**

Orbifold groupoids, sectors, and wreath products

**Abstract:** If  $Q$  is an orbifold represented by an orbifold groupoid  $\mathcal{G}$  and  $\Gamma$  is a finitely generated group, then  $\Gamma$  induces a decomposition of  $Q$  into sectors generalizing the inertia orbifold. Specifically, this sector decomposition is given by the space of groupoid homomorphisms from  $\Gamma$  into  $\mathcal{G}$ , which admits a natural  $\mathcal{G}$ -action. We will discuss invariants of orbifolds associated to these sector decompositions as well as their applications. We will in particular consider the class of wreath symmetric products of orbifolds. This is joint work with Carla Farsi.

**Friday June 5, 9:00 a.m.: Dana Williams, Dartmouth College**

The Mackey Machine for Crossed Products

**Abstract:** In simple terms, "the Mackey Machine" is the process of describing the irreducible representations, or in the non type-I case, the primitive ideals of a crossed product or group  $C^*$ -algebra via induced representations. In 1967, Effros and Hahn conjectured that every primitive ideal of a transformation group  $C^*$ -algebra  $C^*(G, X)$  was induced from a stability group.

This conjecture was verified in 1979 by Gootman and Rosenberg building on work of Sauvageot. However, there remain important questions about exactly when an induced primitive ideal is primitive. In this talk, I will discuss some progress and open questions surrounding this issue, as well as the extension of these ideas to groupoid  $C^*$ -algebras. This is joint work with Siegfried Echterhoff and Marius Ionescu.

**Friday June 5, 10:00 a.m.: Shoichiro Sakai, Nihon University**

Some remarks on the Kadison-Singer problem

**Abstract:** TBA

**Friday June 5, 11:00 a.m.: Nadia S. Larsen, University of Oslo**

Co-universal  $C^*$ -algebras for product systems of Hilbert bimodules

**Abstract:** For each  $X$  in a large class of product systems of Hilbert bimodules we associate a  $C^*$ -algebra which is co-universal for Nica covariant representations of  $X$ . We show that this approach provides a route to proving gauge-invariant uniqueness theorems for  $C^*$ -algebras associated to product systems and gives an efficient tool for studying many examples. This is joint work with T. Carlsen, A. Sims and S. Vittadello.

**Friday June 5, 1:30 p.m.: Liangqing Li, University of Puerto Rico**

On the classification of inductive limit algebras

**Abstract:** In this talk I will report some classification results on the class of simple  $AH$  algebras and  $AH$  algebras with ideal property, The talk is based on joint work of Elliott-Gong-Li and Gong-Jiang-Li-Pasnicu.

**Saturday June 6, 9:00 a.m.: Charles Akemann, University of California at Santa Barbara**

Extending a state from a MASA of a von Neumann algebra

**Abstract:** Ever since the Kadison-Singer paper of 1959, there has been substantial interest in extending pure states from maximal abelian subalgebras (MASAs) of  $C^*$ -algebras. In this talk I will change the problem a bit and ask the following question. Question: Given a MASA  $A$  of a von Neumann algebra  $N$  and a normal state  $g$  of  $A$ , what can be said about the set  $S$  of all state extensions of  $g$  to  $N$ ? There are some large gaps in our knowledge about this question. I will talk about partial results and useful techniques, and I will show how this question relates to various other things like conditional expectations. I promise some interesting unsolved problems.

**Saturday June 6, 10:00 a.m.: Roger Smith, Texas A & M University**

Close separable nuclear  $C^*$ -algebras

**Abstract:** Kadison and Kastler introduced a metric on the set of subalgebras of  $B(H)$  in terms of the Hausdorff distance between unit balls. They conjectured that two sufficiently close  $C^*$ -algebras  $A$  and  $B$  are isomorphic, and further that there is a unitary  $u \in B(H)$  such that  $uAu^* = B$ . Positive resolutions have been obtained for injective von Neumann algebras, and

certain classes of separable nuclear  $C^*$ -algebras, notably the AF-algebras and those of continuous trace. In this talk I will describe the full solution of this problem for separable nuclear  $C^*$ -algebras. Separability is essential here since it is known to be false for nonseparable nuclear  $C^*$ -algebras. This is joint work with Erik Christensen, Allan Sinclair, Stuart White and Wilhelm Winter.

**Saturday June 6, 11:00 a.m.: Hanfeng Li, University of Buffalo**

Turbulence, representations, and trace-preserving actions

**Abstract:** I will give criteria for turbulence in spaces of  $C^*$ -algebra representations, and indicate how this helps to establish results of nonclassifiability by countable structures, for group actions on a standard atomless probability space and on the hyperfinite  $II_1$  factor. This is a joint work with David Kerr and Mikael Pichot.