Teaching Mathematics as if Our Survival Mattered

Martin E. Walter
Department of Mathematics
University of Colorado, Boulder
martin.walter@colorado.edu

December 6, 2012

1 Introduction. As part of the year of mathematics and planet earth, this article began as "mathematics of climate change;" and, in fact, I will spend most of my time presenting some of the basic mathematics, easily accessible to undergraduate and some high school students, which is useful—perhaps indispensable—in understanding climate change. But I now have broadened my scope and briefly mention more examples where mathematics plays a pivotal role in helping us understand "the current human condition." One of my objectives is to establish content similar to that of this article as a new subdiscipline of mathematics—and to establish this subject as a standard part of the undergraduate (even high school) curriculum.

If you find the title of this article a bit melodramatic, that is not my intention. In fact, collapse of contemporary, complex, human civilization might, in a matter-of-fact manner, be expected given the known history of previous civilizations. Clive Ponting, in [12], details the collapse of many earlier civilizations from the environmental/natural resource perspective. Joseph Tainter, [15], chronicles societal collapse via the theory of diminishing marginal returns on investments in complexity. Jared Diamond, in [4], has given a popular account of collapse. He begins his book with a list of "pre-collapse" symptoms concerning the state of modern Montana, including toxic effects of particular mines, forest and agricultural management (or mismanagement), soil and water impacts, invasive species—a list to which I add impacts of asbestos in Libby, Montana, cf., [11]. I thus take it as given that the reader and I share the assumption that collapse is possible, given that it has happened before, for example, to the Romans.

Now my academic friends in biology have told me that a phenomenon basic to their subject is *variation.* I take it as interesting, if not essential, for *any system* in Nature that we study the following three step process: *variation, selection, amplification.* Applied to systems, such as a civilizations, we can recognize variability. Nature tests, selects various systems. Those that pass go on to "amplify." Those that do not "pass the test(s)" might hang on in a greatly reduced form waiting for a new selection process that is more favorable, or perhaps total extinction awaits. Logically this process can be dismissed as trivially tautological. However, it is of great non-tautological interest to estimate the probability that the civilization(s) of which we are a part will pass the tests Nature has in store for us.

I do not believe that mathematics alone will "save us." But I do think that a *mathematical perspective* offers a unique understanding, a foundation stone, an essential piece of the puzzle—increasing the probability that humans will prosper in the future. I might add parenthetically that each of following examples I discuss has

aspects of mathematical interest at a multitude of levels—from the most elementary process of *counting* to the frontiers of current research! I hope that at least a few mathematicians so inclined will join in the fun.

2 A "Law of Gravity" for Global Warming. In a moment I will discuss a "greenhouse-gas law" for carbon dioxide, CO_2; and it does not have much to do with the law of gravity, except that this CO_2-law is as basic to climate science as the law of gravity is to classical physics. In 1824 the French mathematician, Jean Baptiste Joseph Fourier (1768–1830), was likely the first scientist to technically address what we refer to today as the "greenhouse effect," [5, 6]. It is easy to experience this effect, just go inside a glass-enclosed greenhouse (or a vehicle with closed glass windows) while the sun is shining. Today we understand that certain gases in our atmosphere transmit visible spectrum sunlight but act as a barrier/absorber/emitter of infrared spectrum radiation, thus serving as a "blanket" that traps heat on earth. Up to a point this phenomenon is beneficial to human life on earth, but beyond "a certain point" it is not. Understanding this last sentence requires mathematics and science.

Thus in 1896, Swedish scientist, Svante August Arrhenius (1859–1927), 1903 Nobel Prize winner in chemistry, derived mostly from "first principles" the following, cf., [2]:

$$\Delta F = \alpha \; ln(C/C_0), \qquad \text{(Greenhouse Law for } CO_2\text{)}$$

where C is CO_2 concentration measured in parts per million by volume (ppmv); C_0 denotes a baseline concentration of CO_2; α is a constant which the Intergovernmental Panel on Climate Change (IPCC) gives as $\alpha = 6.3 \; \frac{W}{m^2}$; and ΔF is the radiative forcing, measured in Watts per square meter, $\frac{W}{m^2}$, due to the increased (or decreased) value for C, the independent variable. Radiative forcing is directly related to a corresponding (global average) temperature; because by definition radiative forcing is the change in the balance between radiation coming into the atmosphere and radiation going out. A positive radiative forcing tends on average to warm the surface of the Earth, and negative forcing tends on average to cool the surface. (We will not go into the details of the quantitative relationship between radiative forcing and global average temperature.)

Now roughly on average over the surface of the earth the sun provides $240\frac{W}{m^2}$. (Let me insert here where I am getting that number from. On page 69 of [7] we see that if we replaced the earth with a flat disk of the same radius perpendicular to the rays of the sun, the solar power per m^2, called solar flux, on that disk would be $1,372\frac{W}{m^2}$. This is referred to as the solar constant, which varies a bit. Now from solid geometry we know that the area of the earth is 4 times the area of this disk. Thus I claim that the solar flux at the top of the earth's roughly spherical atmospheric surface, averaged over position on the earth, which includes night and day, is $\frac{1,372}{4}\frac{W}{m^2}$. This requires some thought. From [7], page 70, we learn that the earth reflects about 30% of the sun's energy back into space and absorbs about 70%. Thus, $.7 * \frac{1,372}{4}\frac{W}{m^2} \approx 240\frac{W}{m^2}$. It is not clear that we have described what is going on on the surface of the earth, but we will take it as a reasonable approximation for our argument. When I built the solar installation for my place, I took a lot more detailed local information into account!)

We can now understand that the radiative forcing due to increased carbon dioxide represents roughly a 1%, i.e., $\frac{2.28}{240}$, boost in the warming from sunlight, where we took $C = 395$ ppmv and $C_0 = 275$ ppmv, the preindustrial level. Since this boost is constant, anyone who denies that CO_2 contributes to global warming must find some mechanism that cancels this effect—at least on average. For me any debate on climate change must involve this most basic law of Arrhenius. However, I have never seen a public debate that does. I have asked climate scientists why Arrhenius's law is not mentioned in, say, the media discussions of climate; and I got an answer akin to "Americans don't do logarithms."

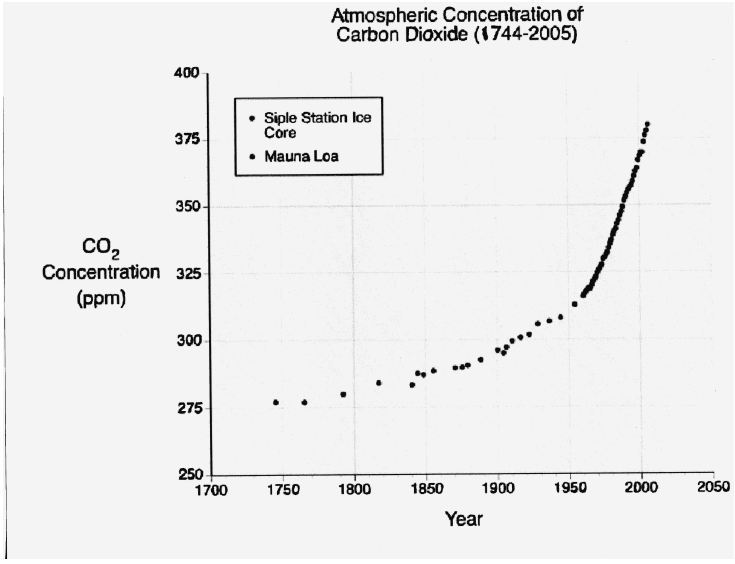

Figure 1: Carbon Dioxide Concentration in the Atmosphere (1744–2005)

At this point one might look for phenomena that mitigate the increasing CO_2 concentrations. Increase in water vapor is often cited as a possibility, for example. As it turns out, invisible water vapor is a potent greenhouse gas. What about clouds? Clouds do reflect sunlight, but they also slow the escape of heat from the planet. A more careful analysis than I can do here indicates that the warming effects of increased water vapor are greater than cooling effects. There are other important greenhouse gases such as methane (which has been escaping from permafrost and industrial agriculture operations), oxides of nitrogen, and others that I have ignored in this article. I have neglected the effects of the increase in fresh water flow from Siberian rivers into the Arctic. I must skip the giant cyclone which replaced the usual anticyclone in the Arctic. Very quickly climate modeling becomes a complex affair requiring a great deal of study. The web site www.realclimate.org, run by actual climate science specialists, and [8], are reasonable places for nonspecialists (such as I) to look for answers to questions they might have.

I have been following the subject of global warming for about twenty years. Climate models have grown ever more detailed and complex; and the computers that run the models have steadily grown larger (in gigabytes) and faster. But CO_2 levels have continued to increase. When there were approximately 6 billion people on earth in the late 1990s humans were putting into the atmosphere about 6 billion tonnes of carbon, C, in the form of CO_2, each year. Thus on average, humans were contributing about one tonne of C per person per year to the earth's atmosphere. As of 2012 the *per capita* emissions have gone up, with more than 7 billion humans emitting. Figure 1 is a graph of atmospheric CO_2 as actually measured, up to 2005. It is now documented that an organized, well-financed effort to disinform the U.S. public about global warming has gone on for quite a while, cf., [17], page 25, while the corresponding problem of ocean acidification has gone largely unnoticed. It turns out that an estimated one-third of *current* CO_2 emissions are absorbed by the world's oceans, while fully half of all fossil carbon emissions released since the beginning of the Industrial Revolution have been so absorbed, cf., [17], page 133. Note that N. Bednarek, et al, in the paper "Extensive dissolution of live pteropods in the Southern Ocean," *Nature Geoscience*, November 25, 2012, report extensive damage to the base of the oceanic food chain due to ocean acidification. This damage has occurred much sooner than many experts expected. Finally, for a unique bit of "data" I recommend James Balog's film, "Chasing Ice." Balog, once a "climate change doubter" now activist, has accomplished the

technically difficult tast of capturing time lapse photoshoots of glaciers in Iceland, Greenland, Montana and Alaska over a period of years. The glaciers are collapsing.

3 Hubbert's Peak, Energy for Civilization, and a Conjecture. There is an interesting bit of mathematics, or more specifically, curve fitting, I want to discuss that has applications to any nonrenewable resource. In 1956 American geophysicist, Marion King Hubbert (1903–1989), predicted that in the early 1970s oil extraction in the United States would finish rising to a peak and then decline thereafter. Hubbert's analysis was rejected at the time he published it, but his prediction came true between 1970 and the Spring of 1971. Hubbert's analysis states that unconstrained extraction of a nonrenewable resource should follow a "bell-shaped curve," reaching its peak when half the resource is exhausted. He stated that the discovery curve would look very much like the extraction curve, but be a translate back in time a fixed number of years—about 40 years for U.S. oil. Figure 2 is a graph of U.S. oil production from 1920 to about 2005.

"Hubbert's Peak" analysis is empirical in nature, and I know of no rigorous proof from first principles. But there is the following heuristic argument. If you have "piles of a nonrenewable resource" of variable quality, buried in various places with variable accessibility, the most easily accessible, highest quality piles will be exploited first. One then proceeds to extract piles which are increasingly difficult to access and of decreasing quality. To get quantitative agreement with real-world data requires the additional mathematical sophistication that Hubbert provided.

So what is the future for fossil fuels, such as coal, oil, natural gas, tar sands, and the like? Climate activists refer to data such as that provided by the Carbon Tracker Initiative, www.carbontracker.org. It is quite likely that the fossil fuel industry's known reserves contain five times the amount of carbon needed to raise global atmospheric/surface temperatures more than 2 degrees C (centigrade) above preindustrial levels. The understanding of climate activists is that such a 2-degree-C rise would create incredible difficulties for human existence. Global temperature is already between .8 and 1 degree C higher than preindustrial levels.

Now the international consensus of climate scientists is represented in reports from the IPCC available at https://www.ipcc.ch/publications_and_data/publications_and_data.shtml. In 2007 the IPCC came out with their Fourth Assessment report, the Fifth is due in 2014. An over-simplified summary of their position is that warming is happening; and it is very likely (greater than 90% chance) that we humans are causing it. Also estimates of between 2 and 4.5 degrees C eventual rise in global temperatures are predicted as possible.

I note, that as to be expected in any system with *variation*, there are those (some with considerable economic and political influence) who are not concerned in the least with the entire topic of climate change and/or even deny that human activity is relevant.

I should mention that the IPCC reports represent an "averaging process." Consider a study that appeared in the November 9, 2012, issue of *Science*, by NCAR (National Center for Atmospheric Resarch) climate scientists John Fasullo and Kevin Trenberth. They evaluated 16 major climate models, all of which were used in the 2007-08 assessment of the IPCC, using a special "relative humidity" metric. (Simply put, their study focused on how accurately climate models measured relative humidity in the subtropics during the dry season.) Those models predicting a 7 degree Fahrenheit increase and above appear to be the most accurate. To quote the authors: "This study does not pin it down. This is just one aspect of things that models need to get right, and if they get it wrong then we don't have confidence in them." More studies evaluating climate models using other important metrics remain to be done. I also take quite seriously the position of climate scientist, James Hansen. Hansen, cf., http://www.youtube.com/watch?v=UcyltBjvyjo, can speak for

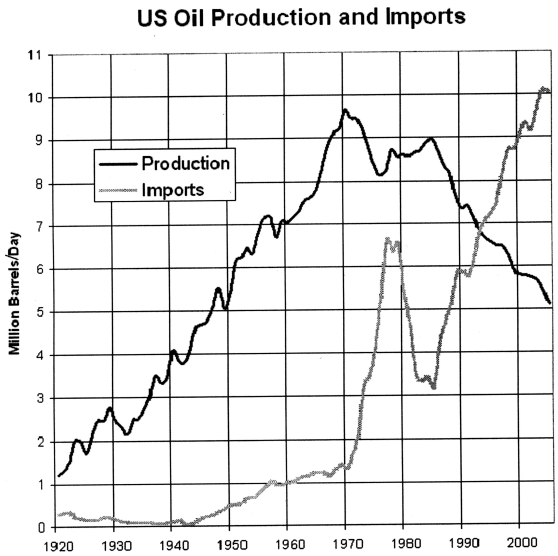

Figure 2: Hubbert's Peak for U.S.A.

himself; but his basic message is that 350 ppmv(CO_2) is the upper bound for a planet liveable in the long term by humans. We are already approaching, and will soon be accelerating past, 400 ppmv (CO_2).

I now would like to make what I call "the wake-up" conjecture: *If we continue business and politics as usual, "the market" will burn enough fossil carbon to completely collapse complex human civilization.*

I will touch on one more bit of support for this conjecture in the next section, but the good news is that we have alternatives. I claim that it is mathematically and economically possible to run the world-wide human economy on renewable energy, cf., [17]. My hope is that if enough people understand this, it will become politically possible. I estimate that it would take a "World War II" level effort, sustained for one or two decades, to make the transition. The result would be a stronger, more durable human economy.

In closing this section I would like to point out that the extraction of any nonrenewable energy and the disposal of resulting waste products creates considerable impacts, which are distributed unequally. Allow me two examples from a list that could easily fill several books. First, industrial natural gas extraction operations "across the street" from homes and schools, prompted the traditional community of Longmont, Colorado, to pass a "ban on hydraulic fracturing within city limits" amendment to its city charter in the November 2012 elections. Such operations are exempted from many federal environmental laws, and a confrontation between the city and the state of Colorado is brewing.

Second, Andrew Nikiforuk, [10], writes about the colossal environmental and social impacts associated with the extraction of oil from Alberta Canada's tar sands. Much of the U.S. Midwest now runs on this source of oil, which contributes three times as much to greenhouse gas emissions as conventional oil, barrel for barrel.

4 Weatherquakes and Mathematics. I have rigorously shown, cf., [16], assuming a reasonable hypothesis, the following theorem (stated qualitatively): *A modest increase in global atmospheric/surface*

temperature, results in a (likely) immodest increase in extreme weather events. I rechristened weather events as *weatherquakes* in analogy with earthquakes. In fact, the mathematics of weatherquake and earthquake distributions are quite similar, with some notable exceptions.

Most of us have heard of the Gutenberg-Richter scale for measuring "the size" of earthquakes. Empirically the number of earthquakes as a function of "this size" obeys a well-known power law; see below for the general form of power laws. The longer the time interval and the larger the geographic area used in the collection of earthquake data, the closer the data fits the aforementioned power law.

Now the power law for earthquakes can be deduced from the following simple hypothesis: Nature does not "prefer" any particular size of earthquake. Nature puts the same amount of "effort" into earthquakes of any particular size on the Richter scale. I then reasoned by analogy. What if there were a way of measuring the "size" of any weather event, i.e., *weatherquake*, and we assumed that Nature did not favor one size over another. This is the essence of my *weatherquake hypothesis*. In conversations with some scientists who are expert in climate science, whether or not they agreed with my conclusions, there was agreement that there is not at this time any known mechanism that would allow for Nature to favor one size of weatherquake over another.

Once the weatherquake hypothesis is admitted, it is only a matter of simple calculus to prove the theorem at the beginning of this section. Coupling this theorem with Arrhenius's Law we get the corollary: *As CO_2 (and other greenhouse gas) concentrations in the atmosphere increase, it is likely that there will be an increase in extreme weather.* Only if the "CO_2 effect," for example, is compensated for in some fashion would this corollary fail; and to date the corollary is consistent with actual data.

$$f[x] = \beta\, x^{\alpha} \qquad \text{(Power Law in General Form)}$$

It follows that the number of weatherquakes as a function of size should follow a power law. Such weather data that I and my former student, Suraje Dessai, have investigated is consistent with the above mathematics; but something I did not anticipate was discovered in the process. Each category of weather event that we studied appeared to obey a power law—but there was a different power law for each category. So, for example, the power law for tornados was different from the power law for hurricanes. Even more surprising was the fact that hurricanes, before the formation of the hurricane "eye," followed one power law; and hurricanes after the formation of the "eye" followed another power law. This interesting complication, however, did not change the above mentioned theorem.

In the past I have heard pundits mock climate science, with comments like: "What's the big deal if the temperature goes up a couple of degrees?" Climate is weather statistics. If we do the math, it apparently is a "big deal," now and then for some people.

A civilization can handle hurricanes, floods, droughts, tornados, blizzards—violent weatherquakes, up to a point. If the weather causes too much damage too frequently, a civilization can fail to keep up with repairs and perish due to exhaustion. This leads to an interesting entropy related calculation, see our last section.

5 Modeling with Spreadsheets. In this and the following sections we mention some topics relevant to the success/failure of human civilizations, our own in particular. We will be quite brief and focus on mathematical or protomathematical content.

First, spreadsheets have applications limited only by one's energy and creativity. For example, I find them particularly useful in modeling population demographics. My first encounter with population modeling was the work of Schwartz, [13], where a spreadsheet was actually a sheet of paper, cf., also [14]. While looking at elementary population models, topics such as doubling time come up very naturally. I find it quite easy to motivate logarithms in this context; and as we have seen in Arrhenius's law, logarithms are important!

My colleague in physics, Professor Albert A. Bartlett, is relatively well-known for his (now video) lecture, cf., www.albartlett.org/presentations/arithmetic_population_energy.html, "Arithmetic, Population, and Energy." One of his famous quotes is: "The greatest shortcoming of the human race is our inability to understand the exponential function." He has tirelessly taught the dangers of exponential population growth; and, indeed, it is hard to think of problems that face humanity that are not made more difficult by rapid growth in human numbers. Given our above discussion I would add to Bartlett's quote: "and the inability of humans to grasp the inverse of the exponential function as well."

Once simple population models are introduced more involved discrete logistic models are close at hand, [17], Section 19.8, which opens the door to the fascinations of many subjects such as chaos theory. One can study epidemics, as in [9]; and then tackle [1].

An entire semester can easily be devoted to "box-flow" models in various contexts, as is done in "Modeling the World in a Spreadsheet: Environmental Simulation on a Microcomputer," [3].

6 The Dunbar Number and Other Limitations of Being Human. Robin Dunbar, cf., [17] Chapter 9, has calculated a number, 147.8, which is an upperbound on the number of people a single person can have in his or her "inner circle" of trusted friends. Some corporations take this number seriously and design "corporate submodules" consisting of teams of employees not to exceed 200. This concept of a Dunbar Number has several possible as well as interesting implications.

Advertising. Experience/history indicates that advertisements and propaganda efforts can be effective. Enormous amounts of money are spent trying to convince people to buy certain things, candidates, or ideas. Messages that occupy our consciousness most can become our "reality." One antidote to this "messaging" is to stop and think through a subject thoroughly. Mathematics is very often a useful tool in this process. Consider the Bagdikian Number (in honor of journalist Ben H. Bagdikian), which I define to the the smallest number of media corporations it takes to own the majority of media, such as newspapers, TV, radio, magazines, books, and so on. When I started thinking about "mathematics for the environment" that number in the U.S. was 50, now it is 6. I claim that this has significantly narrowed the range of debate in the U.S. with negative consequences.

Detachment from Reality. Hunter-gatherers lived "cheek to jowl" with their ambient environment. Since about 8000 B.C. agriculture has transformed human niches in Nature. For the past 200 years with the industrial revolution and urbanization, powered by fossil fuels, many humans have increasingly surrounded themselves with their own "reality." Momentarily feeling free of constraints, we have lost important connections with and daily feedback from Nature. For example, farmers in the dust-bowl days of midwestern America focused on turning prairie sod into wheat fields, ignoring a host of natural variables with disastrous results. In the financial meltdown of 2008, while focused on accumulating money, sound mathematical/financial principles were ignored, again with near system-collapsing results. These and similar examples lead me to conjecture that actually the *greatest failing of humans* is the tendency of people to focus on themselves to the exclusion of other "variables." And society at large does not organize to lessen this tendency, but often

to support it. For example, leading up to the dust bowl the growing of wheat was facilitated in part by the fight to win World War II. In the case of financial collapse, the next one will have been facilitated by our financial support of those who caused this last one—even after they caused it.

The hunter-gatherer option of staying connected with Nature's fundamental variables is closed for, and/or avoided by, most of us. It falls to our educational system, in or out of the classroom, to reconnect civilization with reality—which we ignore even momentarily at our peril. Mathematics can play a key role in this educational process. The simple processes of counting and connecting, for example, can get people thinking about important issues. For example, the Colorado Oil and Gas Association (COGA) stated that $6.5 * 10^{12}$ gallons of water was used in about 43,000 Colorado wells in 2012 to recover fossil fuels. Fossil fuels are used to transport bottled water in petroleum-plastic bottles. In the U.S. we consume roughly $9 * 10^9$ gallons of bottled water a year. COGA states that the amount of water used in their operations is only a fraction of one-percent of the water used in Colorado annually. But that amount is, nevertheless, roughly a 1,000 times our nation's bottled water consumption. And where do plastic bottles end up? They should be 100% recycled as a precious resource, but untold numbers of them end up in the ocean. Countless sea birds, turtles, sea mammals and other sea creatures end up ingesting plastic bags and bits, bottles, lighters (or get entangled in debris)—and die—needlessly. And every day $3 * 10^6$ barrels of water are consumed from the Athabasca River, to produce 10^6 barrels of "tar sands oil." We have had a reasonably stable climate, conducive to agriculture, for the past 10,000 years. Burning fossil carbon has likely destabilized the climate, with negative impacts on agriculture. Keep this counting and connecting exercise going for an other paragraph—as an exercise if you wish.

Speaking of agriculture, in the United States we spend (as an order of magnitude calculation) *10 Calories of fossil fuels to put 1 Calorie of food on our plates.* This last exercise in free association using counting and connecting can and does lead to a host of mathematical modeling. It can be continued indefinitely and can use mathematics at every level of sophistication. The trick is to make sure that the variables/concepts most important to our survival, to our thriving, are not missed.

7 Entropy. I claim that the mathematical/physical concept of entropy is at least as important as energy for understanding such subjects as economics, ecology, social organization, and Nature in general. Both the thermodynamic and information theory (Shannon entropy) formulations of entropy are relevant. Let me illustrate with some nontechnical, qualitative examples. Humans have lived off "low entropy" in the sense that 90% of the "big fish" of the oceans have been taken; as of 1990 96% of "big, old trees" in the U.S. have been taken; and even large schools of smaller fish such as sardines off the coast of California have been taken, cf., Cannery Row. A century and a half ago a miner could find a gold nugget, i.e., an ounce of approximately pure gold. Now many tons of ore need to be processed to recover the same, cf., cyanide heap leaching. Pollution can be viewed as an increase in entropy of the detrimental sort. Extreme weather events such as hurricanes, tornados, floods, droughts usually lead to increased entropy via property destruction, loss of life, dust storms, and fires. Interestingly hurricanes can be viewed as heat engines; and we can easily estimate the mechanical energy generated by a typical hurricane using the second law of thermodynamics, cf., [17], page 544. Global warming leads to increased extreme weather leads to increased disorder. It is interesting to estimate the cost of converting human civilization from fossil carbon to renewable energy over the next one or two decades—and to compare this with the costs of dealing with increased weatherquakes for the foreseeable future.

It is time to wrap up this article, may the adventure of which it is a part continue.

References

[1] Roy M. Anderson, Robert M. May, *Infectious Diseases of Humans*, Oxford University Press, Oxford, New York, Tokyo (1992).

[2] Svante August Arrhenius, "On the Influence of Carbonic Acid in the Air Upon the Temperature of the Ground," *Philosophical Magazine* 1896 (41): pp. 237–76.

[3] Timothy J. Cartwright, *Modeling the World in a Spreadsheet: Environmental Simulation on a Microcomputer*, The Johns Hopkins University Press, Baltimore (1993).

[4] Jared Diamond, *Collapse: How Societies Choose to Fail or Succeed*, Viking Penguin, New York, (2005).

[5] J. B. Joseph Fourier, "Remarques Générales Sur Les Températures Du Globe Terrestre Et Des Espaces Planétaires," *Annales de Chemie et de Physique* (1824) Vol. 27. pp. 136–67.

[6] J. B. Joseph Fourier, "Mémoire Sur Les Températures Du Globe Terrestre Et Des Espaces Planétaires,"*Mémoires de l'Académie Royale des Sciences* (1827) Vol. 7. pp. 569–604

[7] John Harte, *Consider a Spherical Cow: A Course in Environmental Problem Solving*, University Science Books, Sausalito, California, (1988).

[8] William W. Hay, *Experimenting on a Small Planet: A Scholarly Entertainment*, Springer, New York, (2013).

[9] Michael G. Henle, "Forget Not the Lowly Spreadsheet," *The College Mathematics Journal*, 26, No. 4 (September 1995).

[10] Andrew Nikiforuk, *Tar Sands: Dirty Oil and the Future of a Continent*, Greystone Books, Vancouver, Canada (2008).

[11] Andrea Peacock, *Libby, Montana: Asbestos & the Deadly Silence of an American Corporation*, Johnson Books, Boulder, Colorado, (2003).

[12] Clive Ponting, *A New Green History of the World: The Environment and the Collapse of Great Civilizations*, Penguin Books, New York, (2007).

[13] Richard H. Schwartz, "A Simple Mathematical Model for Population Growth," *Journal of Environmental Education*, Vol 12, No. 2. Winter (1980–81).

[14] Richard H. Schwartz, *Mathematics and Global Survival, Fourth Edition*, Ginn Press (1998).

[15] Joseph Tainter, *The Collapse of Complex Societies*, Cambridge University Press, Cambridge, U.K. (1988).

[16] Martin E. Walter, "Earthquakes and Weatherquakes: Mathematics and Climate Change," *Notices of the American Mathematical Society*, Volume 57, Number 10, pp. 1278–1284, November, (2010).

[17] Martin Walter, *Mathematics for the Environment*, Chapman & Hall/CRC, Taylor & Francis Group, New York, (2011).