Earthquakes and Weatherquakes: Mathematics and Climate Change

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In memory of Roger Gallet

In 1824 the French mathematician Jean Baptiste Joseph Fourier (1768-1830) in order to describe certain observations, created the term "greenhouse effect", [7, 8]. In modern language this effect occurs when visible spectrum sunlight passes through an enclosure-creating barrier, such as glass or an atmosphere, and the enclosure heats up because the barrier absorbs/emits infrared spectrum radiation or otherwise traps heat. This paper is one (small) mathematical step on the journey that Fourier began. Our main goal is to describe a plausible model wherein the proportion of extreme weather events, such as tornados, among all weather events, can be expected to increase as the concentrations of greenhouse gases, such as carbon dioxide, increase in the atmosphere.

In 1896 Swedish scientist Svante August Arrhenius (1859-1927), 1903 Nobel Prize winner in chemistry, was aware that atmospheric concentrations of CO_2 (and other gases) had an effect on ground level temperatures; and he formulated a "greenhouse law for CO_2", [1]. Were Arrhenius alive, the motivations for his study and the precise values of physical constants used in his models might change, but his greenhouse law remains intact today. From a reference published about 102 years after [1], namely, page 2718 of [14], we see Arrhenius's greenhouse law for CO_2 stated as:

(Greenhouse Law for CO_2) $\Delta F = \alpha \, ln(C/C_0)$,

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where C is CO_2 concentration measured in parts per million by volume (ppmv); C_0 denotes a baseline or unperturbed concentration of CO_2, and ΔF is the radiative forcing, measured in Watts per square meter, $\frac{W}{m^2}$. The Intergovernmental Panel on Climate Change (IPCC) assigns to the constant α the value 6.3; [14] assigns the value 5.35. Radiative forcing is directly related to a corresponding (global average) temperature, by definition radiative forcing is the change in the balance between radiation coming into the atmosphere and radiation going out. A positive radiative forcing tends on average to warm the surface of the Earth, and negative forcing tends on average to cool the surface. (We will not go into the details of the quantitative relationship between radiative forcing and global average temperature.)

Qualitatively his CO_2 thesis, which Arrhenius was the first to articulate, says: *increasing emissions of* CO_2 *leads to global warming*. Arrhenius predicted that doubling CO_2 concentrations would result in a global average temperature rise of 5 to 6 deg C. In 2007 the IPCC calculated a 2 to 4.5 deg C rise. This is fairly good agreement given that more than a century of technology separates the two sets of numbers.

For the record, cf. [2], preindustrial concentrations of CO_2 are estimated to have been about 280 ppmv. From [2], page 43, we see a table of global, average annual CO_2 concentrations from 1960 when it was 316.91 ppmv to 2006 when it was approximately 381.84 ppmv. In this table the function of CO_2 concentration versus time is essentially an increasing function from 1960 to the present (neglecting the annual fluctuation). We

note that in 2008 CO_2 concentrations of 387 ppmv were measured in Svalbard, Norway.

If we exponentiate both sides of the greenhouse law for CO_2, we obtain a special case of what is referred to as a "power law". Thus

(Power Law in General Form) $\qquad f[x] = \beta x^\alpha$

Let us make some simple observations, which we will use later. If a dependent variable $f[x]$ is a power-law function of (positive) independent variable x, the log-log plot of $\ln f[x]$ versus $\ln x$ (if β is not zero) is a straight line.

Conversely, if a collection of data points (y, x) yields a log-log plot that is a straight line, i.e., the points $(\ln y, \ln x)$ lie at least approximately on a (nonvertical) straight line, then y can be given by a power law in terms of x.

There is also another characterizing property of a power law, namely, *self-similarity* or *independence of scale*. What this means is that if the independent variable is "rescaled", i.e., multiplied by a (positive) constant, the basic form of the relationship is unchanged, i.e., α does not change; only β changes to a different constant. Thus in the case of a set of data points that obey a power law, if the independent variable is rescaled, the straight line log-log graph translates to another straight line parallel to the original, i.e., with the same slope.

Before leaving this introduction, let us return briefly to [14], page 2718. We note that the radiative forcing formula for CO_2 is among the simplest of such formulas for trace greenhouse gases. Other greenhouse gases, such as CH_4 and N_2O, yield radiative forcing functions that are at least superficially far more complex than simple power laws. Note that for methane, CH_4, and oxide of nitrogen, N_2O, the proportionality constants corresponding to the α in the greenhouse law for CO_2 above are much smaller, with estimates varying from .036 to .12; but concentrations of CH_4 and N_2O are measured in parts per billion by volume, ppbv, indicating that these gases are potent even in relatively small concentrations. Chlorofluorocarbons, CFCs, yield a simple linear radiative forcing function, namely, such forcing is a multiple α of changes in concentration measured in ppbv, where values for α vary from .22 to .33. (Radiative forcing, ΔF, is measured in $\frac{W}{m^2}$ throughout.) Finally, we would be remiss if we did not mention that even though water vapor has a residence time in the atmosphere measured in days, cf. [13], page 27, the IPCC has recently stated that its amplifying effects could more than double the warming caused by CO_2 on a global scale.

We close this introduction with related open problems. Can the "weatherquake hypothesis" (stated in the section titled "Weatherquakes and Global Warming/Climate Change") be deduced from accepted basic principles of mathematics,

Assuming that the Californian shocks are representative of general conditions, and attaching the results from California to those found directly for the whole world in the higher magnitude levels, the conclusions follow:

	Magnitude	Annual number
Great earthquakes	8 or more	1
Major earthquakes	7–7.9	10
Destructive shocks	6–6.9	100
Damaging shocks	5–5.9	1000
Minor strong shocks	4–4.9	10000
Generally felt	3–3.9	100000

The total number of shocks potentially strong enough to be perceptible to persons in a settled area (magnitudes 2 and over) must be of the order of several hundred thousand per year. Including aftershocks and swarms of small shocks, the total may be well over a million.

Figure 1. Gutenberg-Richter Table.

physics, and chemistry assuming everything we know about, or assuming a simplified model of, the atmosphere? Can the weatherquake hypothesis be shown to hold for some general class of "self-organizing" systems? Is it provable that the Weatherquake Hypothesis cannot be demonstrated in any of the aforementioned ways?

The Earthquake Distribution and Other Power Laws

Figure 1, reproduced from [12], page 105, displays an amazingly simple set of data, given that it emanates from the complexity of earthquakes.

Again quoting from [12], page 2, we read the following explanation of the *magnitude scale* for earthquakes used by Gutenberg and Richter.

"The magnitude scale number is intended to be logarithmic in the maximum amplitude of earth motion at a fixed distance. The smallest shocks recorded (only on sensitive instruments at a distance of a few kilometers) are of magnitude 0; shocks of magnitude 3 are usually felt; shocks of magnitude $4\frac{1}{2}$ are capable of causing slight damage; major earthquakes range from magnitude 7 to magnitude $8\frac{1}{2}$. Increase in the magnitude by half a unit corresponds to multiplication of the energy released by a factor of 10, so that there is a ratio of about 10^{17} between the energies released in the largest and the smallest earthquakes."

Thus the first column of data in the Gutenberg-Richter Table, Figure 1, is the magnitude of earthquakes. We are told that the numbers in this first column are logarithms (presumably to base 10) of something we shall refer to for simplicity as earthquake *intensity*. If we take the logarithm, to base 10, say, of the second column, which is the number of earthquakes of a given intensity occurring in a given year, we notice a simple invariant. The sum of "log of intensity of event" and "log of the number of events of that intensity (per year)" is approximately constant, namely, 8.

The Gutenberg-Richter data thus yields a log-log plot which is approximately a straight line. This is an empirical fact, i.e., the results of actual measurements of real earthquakes. This (linear) approximation gets better the larger the geographical area involved and the longer the interval of time. There is now an abundance of earthquake data, cf., page 13 of [3], [18], [17], [16], for a few examples, which we take a brief moment to discuss. If one plots the log of N, the number of events of magnitude M, versus M, seismologists traditionally refer to the (negative) of the slope of this line as the "b value." From [17], page 276, we read: "Although the b values approximately equal 1 over long time scales and large spatial scales, significant variations occur on smaller scales." Swarms of seismic events, lacking a main shock, and which may be associated with migration of magmatic fluids, are mentioned as one example. This presents the interesting possibility that subsets of seismic events may be more finely classified according to their b value, and thus the analogy with weather events (discussed in the next section) becomes even closer. Flows of magmatic fluids would intuitively seem more like flows of air/water, than say movements of solid earth crust; although earthquakes and flows of magma are not entirely disassociated.

One must also make the obvious observation that the earthquakes are in some sense bounded above by the size of the earth and below by the size of molecules; thus some aberrations are to be expected in the extremes, although the Gutenberg-Richter law appears to hold even when very small earthquakes are included.

A priori there seems to be no obvious reason why the number of earthquakes of a given intensity is so simply related to the intensity of those earthquakes. Nevertheless, it is an observed fact. The main mathematical point, I repeat, is this: seismic events (classified according to b value if necessary) have associated with them a number, called the *intensity* of the event, such that the log of the number of events of a given intensity versus the log of intensity is (approximately) a straight line. Confronted with such a fact, we try to think of some simple principle that might "explain" it—a simple axiom from which the fact can be deduced.

The mathematics of the situation leads us directly to such an axiom. The Gutenberg-Richter data yields a linear log-log plot that must come from a power law, as explained in the first section of this paper. The numerical values of the slope and intercepts of the line so determined depend on the units chosen to make measurements and the actual values measured. These actual values are of great importance in geology, but to a "pure" mathematician the essential fact is that the log-log plot is a straight line. This fact all by itself tells us that we are dealing with a phenomenon that is *independent of scale*, e.g., a phenomenon described by a power law.

In this particular case we can take the statement (where C is a constant and logs are to base 10):

"log of intensity of event" + "log of the number of events of that intensity (per year)" = C,

and get by exponentiation the equivalent statement:

["intensity of event"] times ["number of events of that intensity (per year)"] = 10^C.

Said informally, our axiom states that there is no preferred "size" or "scale" of earthquake. Nature expends the same total intensity shaking the earth at one point on the Gutenberg-Richter scale as it does at any other point on that scale. The bigger the quakes, the fewer there are of them, and the relationship between number and intensity is fairly precise, namely, the relevant log-log plot is very close to a straight line. It is interesting to note the relationship between energy and intensity, or magnitude. Seismologists estimate, cf., page 273 of [17], [18], the ratio of (radiated) energy expended by earthquakes of Richter magnitude *n+1* divided by (radiated) energy expended by earthquakes of Richter magnitude *n* (for $n \geq 4$) to be $10^{1.5}$, roughly 31.6, or 32. This is close to $10\pi \approx 10^{1.497}$.

Our axiom, which we informally stated above, is thus qualitatively equivalent to the Gutenberg-Richter law, i.e., the table in Figure 1. *Our axiom that nature has no preferred size of earthquake is (qualitatively) equivalent to the appropriate log-log plot being a straight line (without quantitatively specifying precisely the slope and intercepts of that line).*

It is in this qualitative spirit that we ask the question: To what type of probability distribution (or probability density function) does the Gutenberg-Richter law lead? Let z denote earthquake intensity, where $z \geq 1$, then the magnitude x of an earthquake of intensity z is $x = \frac{1}{2}log_{10} z$, where the $\frac{1}{2}$ reflects the historical fact that a $\frac{1}{2}$ increase on the Gutenberg-Richter scale corresponds to multiplying the corresponding earthquake intensity by 10. We can thus write the following power law:

$$N[z] = \beta \, (.1)^{\frac{1}{2} log_{10} z} = \beta \, z^{-\frac{1}{2}},$$

for $z \geq 1$, as qualitatively corresponding to the the Gutenberg-Richter law for earthquakes, where $N[z]$ is the number of earthquakes of intensity z.

Since we are looking for a probability distribution, we change variables from z to x (if for no other reason than that $\int_1^\infty z^{-\frac{1}{2}} dz$ is not finite) and get $N[x] = \beta \, (.1)^x$, for $x \geq 0$. This is a normalized, continuous form of what we will call the geometric probability distribution corresponding to the earthquake data in Figure 1, if $\beta = (\int_0^\infty (.1)^x dx)^{-1} = -ln\,(.1) \approx 2.30259$.

Thus our axiom, that nature has no preferred size (or scale) for earthquakes, implies that earthquakes are geometrically distributed if the number of earthquakes is plotted versus magnitude, i.e., the logarithm of intensity.

I close this section by mentioning that there is a wide variety of natural phenomena described by power laws [3, 15].

Weatherquakes and Global Warming

Climate is, by definition, weather statistics. Thus one might suppose that any number of mathematical tools might be applicable to the study of climate and weather. Dr. Roger Gallet, our friend and a scientist with the National Oceanic and Atmospheric Administration (NOAA) many years ago initiated a sophisticated statistical analysis of weather events using every tool of which we were aware and more, including, for example, Gumbel's work, [11]. He was trying to demonstrate what we refer to as *Gallet's Conjecture*, viz., that the proportion of extreme weather events among all weather events increases as the atmosphere (troposphere) becomes warmer. Our colleague, Dr. Holley briefly joined the effort to statistically verify Gallet's Conjecture. About the time it was becoming evident to us that the results would likely not be definitive, sadly, Dr. Gallet became ill and passed away. Add to this the fact that we were familiar with the now famous work of Jerzy Neyman on the statistics of smoking and health from the last century, and how in the early years it was ignored/attacked by some—with some success since exact mechanisms by which cigarettes impacted health were not then well understood. Any analogous statistical analysis of weather events by this author would likely be greeted with even less enthusiasm. Finally we were (are) of the opinion that should Gallet's Conjecture become statistically, obviously, unassailable, it might be too late to do anything about it. Thus we were motivated to bypass statistics and look for a fundamental mechanism and/or principle (or principles) that would imply the truth of Gallet's Conjecture.

Since power laws appear in such a wide variety of natural phenomena, [3, 15], we investigated the possibility that a power law might find a place in the study of weather events.

We shall use the terms "weatherquake" and "weather event" interchangeably. We can ask: What is the distribution of the number of weather events as a function of event intensity, or as a function of the logarithm of event intensity? It would be easy to wave our hands and say that because weather events are influenced by many small and seemingly unrelated random effects the distribution of weather events should approximate the normal distribution. Of course, we could

have made the same hand-waving argument about earthquakes, which we have seen are distributed geometrically when plotted as a function of the logarithm of event intensity. Ultimately this question is to be answered by empirical observation of weatherquakes, some of which has been done; cf. [4].

The alert reader will have noticed that we have not given a precise definition of weatherquake other than a tautological one. Neither have we given a definition of intensity of a weatherquake or how to go about measuring same. These considerations are actually part of our weatherquake hypothesis.

The Weatherquake Hypothesis

There exists a definition of *weatherquake* and there exists a definition of *intensity of weatherquake* such that nature has no preferred size (or scale) of intensity of weatherquake. Significant nonempty classes of such weatherquakes exist.

Although some statistical measures of hurricanes, for example, are not analogous to that of earthquakes, [6], we have found no arguments supporting the negation of the weatherquake hypothesis, i.e., that there is a reasonable definition of weatherquakes or their intensity such that nature prefers some intensities more than others. Furthermore, one could disprove the weatherquake hypothesis by showing that our conclusion about extreme weatherquakes (in the next section) implied by the weatherquake hypothesis is false. However, empirical evidence thus far is tending to confirm, not contradict, this conclusion [9, 10, 4].

From the point of view of pure mathematics we could remain silent on any proposed definitions of weatherquakes and their intensities, but we owe the reader some discussion of these topics. Thus virtually no one who has been in the presence of a tornado would deny that such is a weatherquake—same for a hurricane. These and some other classes of weatherquake follow power laws [4]. The data tell us that two different classes of weatherquake can (and often do) follow different power laws. This does not affect the conclusion(s) in the next section. In fact, hurricanes switch from one power law to another at eighty-five knots, which coincides with the formation of the hurricane eye.

Thus we claim that the collection of classes of weatherquakes to which the weatherquake hypothesis/power laws apply is not only nonvacuous but socially and scientifically significant. For historical reasons the Saffir-Simpson hurricane wind scale and the Fujita scale of tornado intensity were developed independently and are not directly comparable. There is even a third class of wind weatherquakes varying from light breezes to

high-velocity wind storms (unidirectional) that appear to obey a power law, [4]. Certain collections of precipitation weatherquakes are likely candidates for satisfying the weatherquake hypothesis, and so on. One possibly universal method of defining weatherquakes and measuring their intensity could be this: observe a peak in power, i.e., a peak in energy flow per unit time, in a volume of atmosphere over a given geographical area in a given interval of time. A number of interesting theoretical investigations will suggest themselves to the interested reader—such as comparisons of local maxima of power in subintervals of an interval of time with the maximum over the entire interval, or comparing the total energy of a weatherquake to its peak power, and so on. One must keep in mind, however, practical limitations of the type and number of observations likely to be actually made in the field. We are confident that the weatherquake hypothesis is satisfied, but we do not know the entire collection of weather events to which it applies.

Not everything that happens in the atmosphere is a weatherquake, just as not every activity of the earth's crust is an earthquake. I mention tectonic plate movements, "slow earthquakes", and pyroclastic flows as examples of things that happen (and are often associated with earthquakes) but that are not always in themselves regarded as earthquakes. The same is true of the atmosphere.

The important thing is this: for events in the earth's crust, respectively the earth's atmosphere, there is a significant class, respectively a collection of classes, of quakes to which power laws apply. To these classes the pure mathematics of the next section applies. The model is so simple that little room is left to escape certain conclusions. For a bit more discussion see [4, 19]. We thus proceed to our main conclusion.

Implications of the Weatherquake Hypothesis for Extreme Events

Following the same arguments used in the case of the Gutenberg-Richter law, we see that the weatherquake hypothesis implies (restricting to one type of weatherquake at a time if necessary) that a power law gives the number of weatherquakes as a function of the intensity of said weatherquakes. Also, from the same mathematical argument used in the case of earthquakes, it follows that the number of weatherquakes (of a given type that follow a given power law) plotted versus the magnitude of weatherquake, i.e., logarithm of intensity of weatherquake, is a geometric distribution. Let's suppose that this geometric distribution is $N_p[x] = \beta p^x$, with $0 < p < 1$, where it is easily seen that $\beta = -\ln p$ if $\int_0^\infty N_p[x]\,dx = 1$, i.e., we have a probability distribution. We are abusing terminology slightly because probabilists

refer to this N_p as the probability density function of an exponential random variable if we write $N_p[x] = (-\ln p)e^{(\ln p)x}$, with $x \geq 0$. Below we will use the term *expectation* of N_p, i.e., $E[N_p]$, as probabilists do, namely, $E[N_p] = \int_0^\infty x\,\beta\,p^x dx$.

Thus $N_p[x]$ is the number, actually the number normalized, of weatherquakes of magnitude x, where x is the logarithm of weatherquake intensity. If we define extreme weatherquakes to be those of magnitude $x \geq a$, then $T_p[a] = \int_a^\infty N_p[x]\,dx$ is a measure of the amount of "effort" nature puts into extreme weatherquakes. (This measure can be considered an understated measure, since the horizontal axis, i.e., magnitude, is a logarithm of weatherquake intensity; and the logarithm of intensity increases far more slowly than actual intensity.) The $T_p[a]$ we refer to as the "tail past a" of our distribution. In a real life situation, a is a numerical value indicating a magnitude of weatherquake that no one denies is extreme.

The proof of the theorem below is elementary and is left as an exercise for the reader.

Theorem 1. *Given* $0 < p < 1$ *and the normalized, geometric probability distribution* $N_p[x] = (-\ln p)\,p^x$, *with* $x \geq 0$, *the expectation* $E[N_p]$ *satisfies*

$$E[N_p] = \frac{-1}{\ln p}$$

. *The "tail past* a*" of* N_p, $T_p[a] = -\ln p \int_a^\infty p^x dx$, *satisfies*

$$T_p[a] = p^a.$$

If $0 < p < q < 1$, *then we have the following formula for the fractional increase in the expectation of* N_q *relative to the expectation of* N_p:

$$\frac{E[N_q] - E[N_p]}{E[N_p]} = \frac{\ln p}{\ln q} - 1.$$

We have the following formula for the fractional increase in $T_q[a]$ *over* $T_p[a]$:

$$\frac{T_q[a] - T_p[a]}{T_p[a]} = \left(\frac{q}{p}\right)^a - 1.$$

Let's interpret the above theorem in the context of weatherquakes. If we start with an atmosphere satisfying N_p and then warm it up, i.e., add thermal energy, by the weatherquake hypothesis we should then have an atmosphere satisfying N_q for some q, $0 < q < 1$. Thus there are three choices, $q = p$, $q < p$, or $p < q$. Because we would expect that $E[N_p] < E[N_q]$, this implies that $p < q$.

Now one way to look at this is via the median, i.e., given p, for what value of a is $T_p[a] = \frac{1}{2}$? From Theorem 1 we see that $a = \ln .5/\ln p$. Thus as p increases monotonically in the open interval from 0 to 1, a increases monotonically from 0 to ∞. Thus as p increases nature puts half of its total "effort" into weatherquakes of increasing magnitude.

Hence if there is global warming, e.g., we pass from p to q with $p < q$, it is clear that Theorem 1 predicts an increase in the "effort" nature allocates to extreme weather events. It also appears that "under most circumstances", as p increases to q, the relative increase in $T_p[a]$ is proportionately larger than the relative increase in the expectation, $E[N_p]$. For instance, see the example at the end of this section.

Note that $(\frac{q}{p})^a - 1$ increases as a increases, i.e., the higher the threshold we use to define "extreme weatherquake" the larger the relative fractional increase in the "effort" nature allocates to extreme weatherquakes. It is thus possible to "cheat" by taking "a" sufficiently large for a given p and q to achieve a predetermined conclusion about the relative rise in extreme weatherquakes. Thus if "a is large", a relatively small increase in parameter p (from p to q) yields a modest increase in expectation (from $E[N_p]$ to $E[N_q]$) but a much larger increase in extreme events.

Now let us observe something that may only be interesting in the mathematical sense. There is no way to be sure without finding real empirical data that matches the behavior we are about to describe in this paragraph. (We have not yet found any such data, by the way.) Observe that $p^a \ln p$, for fixed $a > 0$, decreases monotonically from 0, when $p = 0$, to $-(ae)^{-1}$ at $p = e^{-\frac{1}{a}}$, then increases monotonically to 0 at $p = 1$. Thus if a is fixed, i.e., the size of extreme weatherquake is determined, then it is possible to find (infinitely many) examples of $p < q$ such that $p^a \ln p = q^a \ln q$, i.e., the fractional increase in expectation is the same as the fractional increase in the corresponding "tails past a." Just pick $0 < p < e^{-\frac{1}{a}} < q < 1$ suitably. It is also clearly possible to pick p and q in a similar fashion so that the fractional increase in expectation is more than the fractional increase in "tails past a". Of course, q must satisfy $e^{-\frac{1}{a}} < q < 1$, and the options for q become more and more limited the larger a is.

Example. Suppose that weatherquakes are distributed according to $N_{.1}[x] = \beta\,(.1)^x$, where $\beta = -\ln .1 = \ln 10$. The expectation of $N_{.1}$ is $-(\ln .1)^{-1} \approx 0.434294$. Let us choose $a = 1$, i.e., any weatherquake of magnitude $x \geq 1$ is extreme.

Suppose there is a warming and $p = .1$ is replaced by $q = .11$, leading to $N_{.11}[x] = \gamma\,(.11)^x$, where $\gamma = -\ln(.11) \approx 2.20727$. The expectation of $N_{.11}$ is approximately $.453047$, which is about 4.32% greater than the expectation of $N_{.1}$. But the relative increase, $\{T_{.11}[1] - T_{.1}[1]\}/T_{.1}$ is 10%. Of course, the situation is more dramatic for larger a. For example, if $a = 2$, then the 10% becomes 21%, and so on. Note that on a finite earth, if we took $\infty = 10$ in this example, the same conclusion results.

The size of the planet limits the size of weatherquakes, but we note with interest that on December 9, 1997, the Associated Press reported a dust storm covering 20% of Mars.

Summary and Conclusion

People are putting (oxides of) carbon into the atmosphere: more than a ton of carbon per person per year on a global average. The 1896 law of Arrhenius, which has never been repealed, predicts and measurements confirm that warming is occurring. Carbon dioxide deposition in the oceans is lowering their pH (making them more acidic), with possibly dramatic long-term consequences [5].

We have demonstrated in this article that, subject to a reasonable hypothesis, a relatively small increase in global warming leads to larger increases than one might expect in extreme weatherquakes such as tornados, hurricanes, and any other class of weatherquake that satisfies our weatherquake hypothesis. Unanticipated consequences of human activities can sometimes be understood with the help of mathematics, even elementary mathematics, cf., [19].

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