Boulder Probability Seminar on Oct. 13th, 2022

In this lecture I will discuss a remarkable result known as the *Gaussian isoperimetric inequality*. Namely, define the Borel probability measure

$$\gamma(dx) = (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}}\, dx$$

on $\mathbb{R}$, and set $\Phi(t) = \gamma\big((-\infty, t]\big)$ for $t \in \mathbb{R}$. Given a Borel subset $A \subseteq \mathbb{R}^N$ and $t \geq 0$, take

$$A^{(t)} = \{x + h : x \in A \ \&\ h \in \overline{B_{\mathbb{R}^N}(0,1)}\}.$$

Then

$$\gamma^N(A^{(t)}) \geq \Phi\Big(\Phi^{-1}\big(\gamma^N(A)\big) + t\Big).$$

Besides showing that half-spaces are the minimizers for the isoperimetic problem associated with γ^N, this inequality has many applications stemming from its dimension indepedence. In particular, it leads a number of surprising results about Gaussian measures on a Banach space. My goal is to first outline a proof of the inequality and then describe some of its applications in both finite and infinite dimensional settings.