University of Colorado
Department of Mathematics
Problem of the Month
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Show that any finite group G must have an element g such that the elements of the set $G - \{g\}$ can be multiplied in some order, without repetition, to obtain the identity element, 1.

For example, if $G = \{1, a, a^2, a^3\}$ is the cyclic four-element group, then we can take $g = a^2$ and arrange the elements of the set $G - \{g\} = \{1, a, a^3\}$ in the order $(1, a, a^3)$. The product in this order is $1 \cdot a \cdot a^3 = 1$.

If $G = \{1, a, b, c\}$ is the noncyclic four-element group, then we can take $g = 1$ and arrange the elements of the set $G - \{g\} = \{a, b, c\}$ in the order (a, b, c). The product in this order is $a \cdot b \cdot c = 1$