

**University of Colorado**  
**Department of Mathematics**  
**Problem of the Month**  
**October 2019**

Let  $A$  be the set of integers  $n \geq 3$  such that it is possible to construct a regular  $n$ -gon with straightedge and compass. The set  $A$  is infinite, and its first ten elements are 3, 4, 5, 6, 8, 10, 12, 15, 16, 17.

**Question.** What is the natural density of the set  $A$ ?

The **natural density** of a subset  $S \subseteq \{1, 2, 3, \dots\}$  is defined as follows. For each positive integer  $k$ , let  $s(k) = |S \cap \{1, 2, \dots, k\}|$  be the number of elements in the interval  $[1, k]$  that lie in  $S$ . Let  $\Delta(k) = s(k)/k$  be the proportion of numbers in the interval  $[1, k]$  that lie in  $S$ . The natural density of  $S$  is  $\lim_{k \rightarrow \infty} \Delta(k)$ , if this number exists.