

University of Colorado
Department of Mathematics
Problem of the Month
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The number of divisors of a positive integer n is denoted $\sigma(n)$.

Suppose that $a_0 < a_1 < a_2 < \cdots$ is an infinite increasing sequence of positive integers with the property that $\sigma(a_i) = \sigma(a_j)$ for all i and j . Show that there exists a positive integer d and an infinite subsequence $a_{n_0} < a_{n_1} < \cdots$ such that $\gcd(a_{n_i}, a_{n_j}) = d$ for all i and j .