

University of Colorado
Department of Mathematics
Problem of the Month
November 2019

Let $g \geq 0$ be a continuous function on $[0, \infty)$ that is monotone nonincreasing. Assume that $\int_0^\infty g(x) dx < \infty$. Prove that for any continuous function f on $[0, \infty)$ satisfying $|f| \leq g$, one has

$$\lim_{h \rightarrow 0^+} h \sum_{n=1}^{\infty} f(nh) = \int_0^\infty f(x) dx.$$