If $M \in M_{2 \times 2}(\mathbb{Z})$ is a $2 \times 2$ integer matrix, then by a *cycle of length $n$ for $M$* we mean a nonzero $2 \times 1$ vector $v$ with integer entries such that

$$v, \ Mv, \ M^2v, \ldots, \ M^nv = v$$

are $n$ distinct vectors that $M$ permutes in a cycle.

What are the possible lengths of cycles for matrices in $M_{2 \times 2}(\mathbb{Z})$?