

University of Colorado
Department of Mathematics
Problem of the Month
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If $M \in M_{2 \times 2}(\mathbb{Z})$ is a 2×2 integer matrix, then by a *cycle of length n for M* we mean a nonzero 2×1 vector \mathbf{v} with integer entries such that

$$\mathbf{v}, M\mathbf{v}, M^2\mathbf{v}, \dots, M^n\mathbf{v} = \mathbf{v}$$

are n distinct vectors that M permutes in a cycle.

What are the possible lengths of cycles for matrices in $M_{2 \times 2}(\mathbb{Z})$?