What is the largest constant $C > 0$ for which the following holds? If $f$ is continuously differentiable on the unit interval with $f(0) = f(1) = 0$, then

$$\max_{[0,1]} |f'(x)| \geq C \int_0^1 |f(x)| \, dx.$$