University of Colorado
Department of Mathematics

Problem of the Month

March 2019

For a function $g : \mathbb{R} \to \mathbb{R}$, let $\|g\|$ mean $\sup_{x\in\mathbb{R}}|g(x)|$. If the function g is unbounded on $\mathbb{R}$, write $\|g\| = \infty$.

Assuming that $r \cdot \infty = \infty \cdot r = \infty \cdot \infty = \infty$ if necessary, show that if $f : \mathbb{R} \to \mathbb{R}$ is twice continuously differentiable, then

$$\|f'\|^2 \leq 2 \cdot \|f\| \cdot \|f''\|.$$