

**University of Colorado**  
**Department of Mathematics**  
**Problem of the Month**  
**February 2020**

Suppose that a finite group  $G$  has the property that there exists a positive integer  $n$  such that for any two elements  $x, y \in G$  the following three equations hold:

$$(xy)^n = x^n y^n, \quad (xy)^{n+1} = x^{n+1} y^{n+1}, \quad \text{and} \quad (xy)^{n+2} = x^{n+2} y^{n+2}.$$

Show that  $G$  must be commutative.