

University of Colorado
Department of Mathematics
Problem of the Month
December 2019-January 2020

Two functions $f, g : \mathbb{R} \rightarrow \mathbb{R}$ commute if $f(g(x)) = g(f(x))$ holds for every $x \in \mathbb{R}$.

- (1) Show that for any $g : \mathbb{R} \rightarrow \mathbb{R}$ there is a function f , which is not the identity function, that commutes with g .
- (2) Show that there exist two functions $g, h : \mathbb{R} \rightarrow \mathbb{R}$ such that any function f that commutes with both g and h must be the identity function.