University of Colorado Department of Mathematics Problem of the Month December 2016

Define a sequence of positive integers as follows: $a_0 = 1$, and a_{n+1} is the least prime number strictly larger than $a_0 + a_1 + \cdots + a_n$. The sequence starts out

$$(a_0, a_1, a_2, \ldots) = (1, 2, 5, 11, 23, \ldots).$$

Show that the polynomial $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$ is irreducible over \mathbb{Q} .