Topology and Geometry Prelim Syllabus
Department of Mathematics
University of Colorado

Topology

Point-Set Topology. Topological spaces, countability axioms, product topology, box topology, subspace topology, quotient topology, connectedness, separation axioms (T0, T1, T2/Hausdorff, T3, T4), compactness, one point compactifications, Tychonoff's theorem.

Fundamental Groups and Homotopy. Definition of the fundamental group, Seifert-van Kampen theorem, simply connected spaces, deformation retractions, fundamental groups of circle, spheres and compact surfaces, both orientable and non-orientable, functorial properties of the fundamental group, applications of the fundamental group: Brouwer fixed-point theorem, fundamental theorem of algebra, Hairy Ball theorem

Covering Spaces. Basic properties, existence of universal covering spaces, example of the circle, lifting lemma, covering homotopy lemma, properly discountinuous actions of groups (on simply connected spaces).

Differential Geometry

Smooth Manifolds. Implicit function theorem and regular values of functions, coordinate charts on manifolds, topological and C^∞ manifolds, examples of manifolds: Lie groups, submanifolds defined via implicit functions, and the classification of two-dimensional compact manifolds.

Vector Fields and Tangent Bundle. Tangent spaces and vectors as differential operators, the tangent bundle over a manifold and triviality, vector fields on manifolds, their local flows, and straightening of vector fields, the Lie bracket of vector fields, as a commutator and as the derivative of a field in the direction of a flow.

Differential Forms and Tensor Fields. Dual spaces and multilinear operators, k-forms on a vector space and wedge products, k-forms and other tensor fields on a manifold, change-of-coordinate formulas for vector fields and k-forms, the d operator from k-forms to $(k+1)$-forms, k-chains and Stokes' theorem on manifolds.

Riemannian Manifolds. Riemannian metrics on manifolds, covariant derivatives and the Levi-Civita connection, parallel transport and the geodesic equation, the Riemann curvature tensor and sectional curvature.