RETURN THIS COVER SHEET WITH YOUR EXAM AND SOLUTIONS!

Analysis

Ph.D. Preliminary Exam
Department of Mathematics
University of Colorado Boulder

January, 2013

INSTRUCTIONS:

1. Answer each of the six questions on a separate page. Turn in a page for each problem even if you cannot do the problem.

2. Label each answer sheet with the problem number.

3. Put your number, not your name, in the upper right hand corner of each page.

Analysis prelim questions, 2012/2013 December 17, 2012

1. Let $f \in L^\infty([0,1])$, $f \neq 0$. Show that the limit
$$\lim_{p\to\infty} \frac{\int_0^1 |f|^{p+1} dx}{\int_0^1 |f|^p dx}.$$
exists and compute it.
2. Is it true that for any $f \in L^1([0,1])$ there exists $[a,b] \subset [0,1]$, $a < b$, such that $f \in L^2([a,b])$?
3. Let $E \subset [0,1]$ denote the set of all numbers x that have some decimal expansion $x = 0.a_1a_2a_3...$ with an $a_n \neq 2$ for all n. Show that E is a measurable set, and calculate its measure.
4. Show that if $A_n \subset [0,1]$ and Lebesgue-measurable, with measure at least $c > 0$ for each $n \geq 1$, then the set of points which belong to infinitely many sets is measurable and its measure is at least c.
5. Construct Lebesgue-measurable real valued functions on $[a,b]$ so that they converge to zero pointwise but there is no null set N in $[a,b]$ such that the convergence is uniform outside of N. (I.e. Egoroff's Theorem is sharp.)
6. Prove that for any function $f : \mathbb{R} \to \mathbb{R}$, the set of its continuity points is G_δ.