MATH 2400: CALCULUS 3

10:30 am - 1pm, Wed. May 4, 2016

FINAL EXAM

I have neither given nor received aid on this exam.

Name:____

Check one below !

◯ **001** WATTS ....(9AM)
◯ **002** GREEN .... (10AM)
◯ **003** BLAKESTAD .... (11AM)
◯ **004** MISHEV .... (12PM)
◯ **005** WASHABAUGH .... (1PM)
◯ **006** BULIN .... (2PM)
◯ **007** CHHAY .... (3PM)

Notes, electronic devices, and any other aids are **not** permitted on this exam.

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **logical**, **legible**, and **correct**. Show all of your work, and give adequate explanations. No shown work even with the correct final answer will lead to no points. Only give one answer to each problem! If there are two different answers to one problem, the lower score will be chosen.

DO NOT WRITE IN THIS BOX!

Problem	**Points**	**Score**
1	12 pts	
2	13 pts	
3	12 pts	
4	13 pts	
5	12 pts	
6	12 pts	
7	13 pts	
8	13 pts	
TOTAL	100 pts	

1. **(12 points)**

The following questions are true/false or multiple-choice. No partial credit will be given and no work is required to be shown on this problem only. Circle your answer.

(a) Let $\vec{v}$ and $\vec{w}$ be two nonzero and nonparallel vectors. Let θ be the angle between $\vec{v}$ and $\vec{w}$ with $0 < \theta < \pi$. Then the area of the parallelogram determined by $\vec{v}$ and $\vec{w}$ is given by

(i) $\vec{v} \times \vec{w}$ (ii) $\|\vec{v}\|\|\vec{w}\| \cos\theta$ (iii) $\|\vec{v}\|\|\vec{w}\| \sin\theta$

(b) Two nonzero vectors $\vec{v}$ and $\vec{w}$ are perpendicular if and only if $\vec{v} \cdot \vec{w} = 0$.

(i) True (ii) False

(c) If f is a continuous function on a closed and bounded set D in $\mathbb{R}^2$, then f attains both an absolute maximum value and an absolute minimum value on D.

(i) True (ii) False

(d) If $\vec{F}$ is a vector field in space whose components have continuous second-order partial derivatives, then $\nabla(\operatorname{curl} \vec{F})$

(i) does not make sense (ii) makes sense and is always zero
(iii) makes sense and may be nonzero

(e) If f is a function of three variables that has continuous second-order partial derivatives, then $\operatorname{curl}(\nabla f)$

(i) does not make sense (ii) makes sense and is always zero
(iii) makes sense and may be nonzero

(f) If f is a function of three variables that has continuous second-order partial derivatives, then $\operatorname{div}(\nabla f)$

(i) does not make sense (ii) makes sense and is always zero
(iii) makes sense and may be nonzero

2. **(13 points)** If the limit exists, evaluate it. If not, explain why not.

(a) **(5 points)**

$$\lim_{(x,y)\to(1,2)} \frac{y^3 - 4x}{x^3 + 4y^3}$$

(b) **(8 points)**

$$\lim_{(x,y)\to(0,0)} \frac{2x^4 - 3y^4}{x^4 + y^4}$$

3. **(12 points)**

Let S be the surface

$$\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{2} = 1.$$

(a) **(7 points)** Find the equation of the tangent plane to the surface S at the point $(1, 0, -1)$.

(b) **(5 points)** Find the angle between the tangent plane in part (a) and the plane $y - z = 1$.

4. **(13 points)** Find and classify as local maximum, local minimum, or saddle point the critical points of

$$f(x, y) = x^3 - 2xy + \frac{y^2}{2}.$$

5. **(12 points)** Compute $\iint_M (x+y)dS$ where M is the part of the plane $x+y+z=2$ in the first octant.

6. **(12 points)** Consider the vector field $\mathbf{F}(x, y) = \langle y^2, 2xy + x \rangle$.

(a) **(5 points)** Is $\mathbf{F}$ conservative? If yes, find a potential function f for it. If not, justify your answer.

(b) **(7 points)** Let C be the positively oriented boundary of the triangle with vertices $(0, 0)$, $(0, 1)$ and $(-1, 0)$. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C y^2 \, dx + (2xy + x) \, dy.$$

7. (13 points) Let $\mathbf{F}(x, y, z)$ be the vector field

$$\langle xz - y^3 \cos(z),\ x^3 e^z,\ z e^{x^2+y^2+z^2} \rangle.$$

Find the flux of the **curl** of $\mathbf{F}(x, y, z)$ across the upper hemisphere of $x^2 + y^2 + z^2 = 1$, oriented upwards. (Use Stokes' Theorem to replace the surface with an easier surface.)

8. **(13 points)** Let S be the box with faces $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 3$, and $z = 5$, with each face oriented outwards. Compute the integral

$$\iint_S \langle x^4y^2z,\ x^3y^3z,\ x^3y^2z^2\rangle \cdot d\mathbf{S}.$$