MATH 2400: CALCULUS 3

5:15 - 6:45 pm, Mon. Feb. 8, 2016

MIDTERM 1

I have neither given nor received aid on this exam.

Name:_____

Check one below !

- ◯ **001** Watts ..... (9am)
- ◯ **002** Green ..... (10am)
- ◯ **003** Blakestad ..... (11am)
- ◯ **004** Mishev ..... (12pm)
- ◯ **005** Washabaugh ..... (1pm)
- ◯ **006** Bulin ..... (2pm)
- ◯ **007** Chhay ..... (3pm)

Notes, electronic devices, and any other aids are **not** permitted on this exam.

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **logical**, **legible**, and **correct**. Show all of your work, and give adequate explanations. No shown work even with the correct final answer will lead to no points. Only give one answer to each problem! If there are two different answers to one problem, the lower score will be chosen.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	17 pts	
2	17 pts	
3	16 pts	
4	17 pts	
5	16 pts	
6	17 pts	
TOTAL	100 pts	

1. (17 points)

(a) **(12 points)** Find an equation of the plane that passes through the three points $P = (1, 1, 0)$, $Q = (0, 2, 1)$, and $R = (3, 2, -1)$.

(b) **(5 points)** Give the parametric equations of the line perpendicular to the plane from part (a) that passes through P.

2. (17 points)

(a) **(12 points)** Consider the surface S given by the equation

$$x^2 - 2y^2 + z^2 = 1.$$

Sktech the intersection of S with the planes

i. $x = 0$
ii. $y = 0$
iii. $z = 0$
iv. $z = 2$
v. $x = 1$
vi. $x = 2$

Remember to label your axes!

(b) **5 points** Write down the equation of the paraboloid with apex at $(0, 0, 0)$ opening in the positive x-direction which intersects the plane $x = 4$ in a circle of radius 3.

3. (16 points) Let ρ be the plane given by the equation $x + 2y + 3z = 6$ and let L be the line passing through the points $P(1, 0, 0)$ and$Q(-1, 3, 1)$.

(a) **(8 points)** What is the intersection of L and ρ?

(b) **(8 points)** Compute the angle between L and ρ.

4. (17 points)

(a) **(9 points)** Two particles travel along the space curves $\vec{r}_1 = \langle 2t, -t^3, 2t^2 \rangle$ and $\vec{r}_2 = \langle 1 + t, -t^2, 1 - t \rangle$. Do the particles collide? If so, find the coordinates of the point of collision. If not, explain why they do not collide.

(b) **(8 points)** Find parametric equations for the tangent line to the curve with parametric equations

$$x = t + \sin t, \quad y = t - \cos t, \quad z = te^{-t}$$

at the point when $t = 0$.

5. (16 points)

(a) **(6 points)** Find the cylindrical coordinates of the point P given by spherical coordinates $(4,\ 3\pi/4,\ 2\pi/3)$.

(b) **(10 points)** The solid E lies strictly inside the sphere $x^2+y^2+z^2=4$ and strictly below the cone $z=-\sqrt{\frac{1}{3}(x^2+y^2)}$. Describe E using inequalities in spherical coordinates, and simplify your answer as much as possible. [Hint: you may find it helpful to consider the traces $y=0$ and $z=-1$.]

6. (17 points) Let $\mathbf{r}(t) = \left\langle \frac{1}{2}e^t(\cos(t) + \sin(t)),\ \frac{1}{2}e^t(\cos(t) - \sin(t)) \right\rangle$.

(a) **(7 points)** What is the arclength of $\mathbf{r}(t)$ between $t = 0$ and $t = 1$?

(b) **(7 points)** What is the curvature of $\mathbf{r}(t)$?

(c) **(3 points)** What is the radius of the osculating circle at $t = 1$?