MATH 2400: CALCULUS 3

7:30 - 10:00 am, Tues. Dec. 15, 2015

FINAL EXAM

I have neither given nor received aid on this exam.

Name:____ **Cancelled due to snowstorm and never given**

Check one below !

- ◯ **001** Bulin .... (9am)
- ◯ **002** Molcho .... (10am)
- ◯ **003** Ih .... (11am)
- ◯ **004** Spina .... (12pm)
- ◯ **005** Spina .... (1pm)
- ◯ **006** Preston .... (2pm)
- ◯ **007** Preston .... (3pm)
- ◯ **008** Chhay .... (9am)
- ◯ **009** Walter .... (11am)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **logical**, **legible**, and **correct**. Show all of your work, and give adequate explanations. No shown work (even with the correct final answer), no points! Only one answer to each problem! In case of two different answers to one problem, the lower score will be chosen!

In case of any need of scratch paper, use the backsides instead of extra sheets and in the problem(s) clearly indicate where your solutions are located.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	12 pts	
2	13 pts	
3	12 pts	
4	13 pts	
5	12 pts	
6	12 pts	
7	13 pts	
8	13 pts	
TOTAL	100 pts	

1. **(12 points)**

The following questions are true/false or multiple-choice. No partial credit will be given and no work is required to be shown on this problem only. Circle your answer.

(a) If f is a function, then $\operatorname{div}(\nabla f)$

(i) does not make sense (ii) makes sense and is always zero (iii) makes sense and may be nonzero

(b) If $\mathbf{F}$ is a vector field, then $\operatorname{curl}(\operatorname{div}\mathbf{F})$

(i) does not make sense (ii) makes sense and is always zero (iii) makes sense and may be nonzero

(c) If $\mathbf{F}$ is a vector field, then $\operatorname{div}(\operatorname{curl}\mathbf{F})$

(i) does not make sense (ii) makes sense and is always zero (iii) makes sense and may be nonzero

(d) If $f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$, then defining $f(0,0) = 0$ makes f continuous.

(i) True (ii) False

(e) If $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors with $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is either 0 or π in radians.

(i) True (ii) False

(f) The osculating plane to a curve at a point is perpendicular to the vector

(i) Tangent $\mathbf{T}$ (ii) Normal $\mathbf{N}$ (iii) Binormal $\mathbf{B}$

2. **(13 points)** You are standing on a mountain with shape described by the equation $z = f(x, y) = 100 - 3x^2 - 2y^2$ at the point $(1, 2, 89)$.

(a) **(4 points)** In what unit direction should you move to descend as quickly as possible?

(b) **(3 points)** What is the rate of change of z in this direction from part (a)?

2. (continued from previous page)

(c) **(4 points)** In what two unit directions can you move to stay at constant height $z = 89$?

(d) **(2 points)** What is the rate of change in the directions from part (c)?

3. **(12 points)**

Consider the surface S given by $z = 2x^2 - 2x^2y + y - 1$.

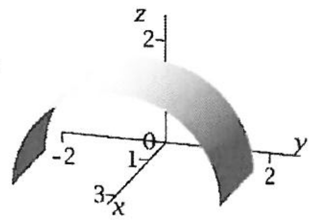
(a) **(7 points)** Find the equation of the tangent plane at the point $(1, 1, 0)$.

(b) **(5 points)** Find the angle between the tangent plane from part (a) and the xy-plane.

4. **(13 points)** Find the point where the absolute (global) maximum of the function $f(x, y) = x^2 + y^2 - 2x$ occurs on the set $x^2 + y^2 \leq 9$. Show all work.

5. **(12 points)**

Compute $\iint_S (z + xy)\, dS$, where S is the part of the cylinder $y^2 + z^2 = 4$ lying between the planes $x = 1$ and $x = 3$ and above the plane $z = 0$.



6. **(12 points)** Let $\mathbf{F}(x, y, z) = \langle \sin y, x\cos y + z, y - \cos z \rangle$.

(a) Show that F is conservative by finding a potential function for it.

(b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ if C is the curve $\mathbf{r}(t) = \langle t, \pi t^2, \pi t^3 \rangle$ for $0 \leq t \leq 1$.

7. (13 points) Let S be the surface given by the function $z = x^2$, above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$. Let $\mathbf{F}(x, y, z) = \langle xy, y + z, 2z \rangle$. Compute the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ along the boundary curve C of S, oriented counterclockwise when looking down on it. (Hint: Use Stokes' Theorem.)

8. **(13 points)** Let E be the region in the first octant bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 2$, and let S be its boundary surface with outward orientation.

(a) **(4 points)** Draw a picture of E and S, and write the unit normals for each of the four surfaces.

(b) **(5 points)** If $\mathbf{F} = \langle x, -y, z \rangle$, compute the flux $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ directly. (Hint: some terms are zero.)

8. (continued from previous page)

(c) **(4 points)** Compute the same flux from part (b) using the Divergence Theorem.