MATH 2400: CALCULUS 3

5:15 - 6:45 pm, Mon. Nov. 16, 2015

MIDTERM 3

I have neither given nor received aid on this exam.

Name:____

Check one below !

- ◯ **001** Bulin .... (9am)
- ◯ **002** Molcho .... (10am)
- ◯ **003** Ih .... (11am)
- ◯ **004** Spina .... (12pm)
- ◯ **005** Spina .... (1pm)
- ◯ **006** Preston .... (2pm)
- ◯ **007** Preston .... (3pm)
- ◯ **008** Chhay .... (9am)
- ◯ **009** Walter .... (11am)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **logical**, **legible**, and **correct**. Show all of your work, and give adequate explanations. No shown work even with the correct final answer, no points ! Only one answer to each problem ! In case of two different answers to one problem, the lower score will be chosen !

In case of any need of scratch paper, use the backsides instead of extra sheets and in the problem(s) clearly indicate where your solutions are located.

DO NOT WRITE IN THIS BOX!

Problem	Points	Score
1	16 pts	
2	16 pts	
3	17 pts	
4	17 pts	
5	17 pts	
6	17 pts	
TOTAL	100 pts	

1. (16 points) Let E be the solid bounded by the planes

$$x = 0; \quad y = 0; \quad z = 0, \quad \text{and} \quad 3x + 2y + z = 6.$$

Suppose that it has density function $\rho(x, y, z) = x$. Then find the mass of E.

2. (16 points) Let C be the wire given by the helix

$$x = 2\cos t; \quad y = 2\sin t; \quad z = 3t,$$

where $0 \le t \le 2\pi$. Suppose that it has density function $\rho(x, y, z) = z$. Then find $\overline{z}$ (= the z-coordinate of the center of mass of C).

3. (17 points) Compute the surface area of the part of the paraboloid

$$z = 16 - x^2 - y^2$$

that lies above the xy-plane.

4. (17 points) Consider the vector field

$$\mathbf{F}(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}.$$

(a) If it is conservative, then find all (real-valued) functions $f(x, y)$ such that $\mathbf{F} = \nabla f$. Otherwise, tell why not.

(b) Suppose that C is the closed curve on the xy-plane that is formed by the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$ and that is traversed in that order. Then evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{r}(t)$ is a vector function giving C.

5. (17 points) Consider the triangle C consisting of the line segments from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(0,1)$, and from $(0,1)$ to $(0,0)$. Evaluate

$$\int_C x\,dx + xy\,dy$$

in two ways: (a) directly and (b) via Green's theorem.

(a) Directly

(b) Via Green's theorem

6. (17 points) Let R be the region on the xy-plane bounded by the following curves

$$xy = 1; \quad xy = 3; \quad \frac{y}{x^2} = 2; \quad \frac{y}{x^2} = 5.$$

Find the area of R. (*Hint.* A change of variables may be useful.)