MATH 2400: Calculus 3, Spring 2014
Midterm 2

March 5, 2014

NAME:

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

Circle your section.

001 J. Migler .... (9am)

002 T. Davison .... (10am)

003 I. Mishev .... (11am)

004 I. Mishev .... (12pm)

005 M. Walter .... (1pm)

006 S. Andrews .... (2pm)

You must show all of your work. Please write legibly and box your answers. The use of calculators, books, notes, etc. is not permitted on this exam. Please provide exact answers when possible. For example, if the answer is π, write the symbol "π" and not the decimal 3.14159. . . .

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. Let the surface S be the graph of $f(x, y) = \ln(1 + x - y)$.
   (a) (6 points) Find a vector normal to S at the point $(0, 0, 0)$ (hint: consider S as a level surface of a function).

   (b) (7 points) Find the equation of the tangent plane to S at the point $(0, 0, 0)$.

   (c) (7 points) Find the quadratic Taylor polynomial $Q(x, y)$ for the function f about the point $(0, 0)$.

2. Suppose that $z = F(x, y)$ and that $x = X(u, w)$ and $y = Y(u, w)$, where F, X and Y all have continuous partial derivatives at all points.

   Caution: one can view z as a function of x and y, and one can view z as a function of u and w.

   Suppose that the following facts are given:

$$\begin{array}{llll}
\dfrac{\partial z}{\partial x}(3,4) = q & \dfrac{\partial z}{\partial x}(a,b) = 5 & \dfrac{\partial z}{\partial y}(3,4) = 11 & \dfrac{\partial z}{\partial y}(a,b) = 12 \\[2ex]
\dfrac{\partial x}{\partial u}(a,b) = 10 & \dfrac{\partial x}{\partial w}(a,b) = 7 & \dfrac{\partial y}{\partial u}(a,b) = p & \dfrac{\partial z}{\partial w}(a,b) = 0 \\[2ex]
X(p,q) = -1 & Y(p,q) = -7 & X(a,b) = 3 & Y(a,b) = 4
\end{array}$$

   (a) (10 points) Find $\dfrac{\partial z}{\partial u}(a, b)$

   (b) (10 points) Find $\dfrac{\partial y}{\partial w}(a, b)$

3. Let

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) (5 points) Using the limit definition, calculate $f_x(0,0)$.

(b) (5 points) Using the limit definition, calculate $f_y(0,0)$.

(c) (5 points) Using the limit definition, calculate $f_{\vec{u}}(0,0)$, where $\vec{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

(d) (5 points) Is f differentiable at $(0,0)$? Justify your answer (hint: compare your answer to (c) with $\text{grad} f(0,0) \cdot \vec{u}$).

4. (a) (10 points) Evaluate the integral:

$$\int_0^1 \int_0^1 ye^{xy} dxdy.$$

(b) (10 points) Evaluate the integral by switching the order of integration:

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2+x^3} dxdy.$$

5. Consider the function $f(x, y) = 2x^3 + 3y^2 - 6xy + 7$.
   (a) (8 points) Find all the critical points of f.

   (b) (8 points) Classify each critical point as a local maximum, local minimum, or saddle point.

   (c) (4 points) Does f have a global maximum on $\mathbb{R}^2$ (i.e. the plane)? How about a global minimum?