Name: ________________________ **Class:** ___________________ **Date:** __________ **ID: A**

Final Exam Review

Short Answer

1. Find the distance between the sphere $(x-1)^2+(y+1)^2+z^2=\frac{1}{4}$ and the sphere $(x-3)^2+(y+2)^2+(z+2)^2=1$

2. Find $|\mathbf{a}|$, $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a}+4\mathbf{b}$ given $\mathbf{a}=\langle \mathbf{3},\mathbf{2},-\mathbf{1}\rangle$ and $\mathbf{b}=\langle \mathbf{0},\mathbf{6},\mathbf{7}\rangle$.

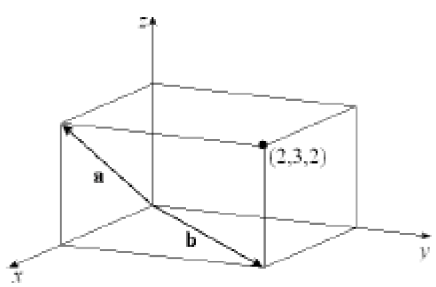
3. Given $\mathbf{a}=\langle 1,1\rangle$, $\mathbf{b}=\langle -4,2\rangle$, and $\mathbf{c}=\langle 5,2\rangle$, find s and t such that $\mathbf{c}=s\mathbf{a}+t\mathbf{b}$.

4. Let $|\mathbf{a}|=1$ and $|\mathbf{b}|=2$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ be 150°. Find

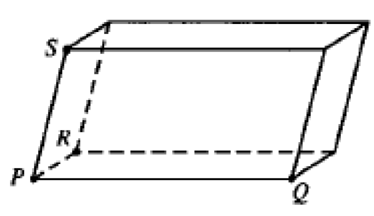
   (i) $\mathbf{a}\cdot\mathbf{b}$ (ii) $\mathbf{b}\cdot(3\mathbf{a}+\mathbf{b})$
   (iii) $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$ (iv) $|2\mathbf{a}-\mathbf{b}|^2$

5. Let $\mathbf{a}=\langle -1,2,2\rangle$ and suppose that $\mathbf{b}$ is a vector parallel to $\mathbf{a}$. Find $\mathrm{proj}_{\mathbf{b}}\,\mathbf{a}$.

6. Find $\mathbf{a}\times\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are given in the figure.



7. Find the volume of the parallelepiped below given $P=(1,-3,2)$, $Q=(3,-1,3)$, $R=(2,1,-4)$, and $S=(-1,2,1)$.



8. Compute $|\mathbf{a}\times\mathbf{b}|$ if $|\mathbf{a}|=3$, $|\mathbf{b}|=7$, and $\mathbf{a}\cdot\mathbf{b}=\mathbf{0}$.

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9. Find symmetric equations of the line passing through (2, –3, 4) and parallel to the vector **AB**, where A and B are the points (–2, 1, 1) and (0, 2, 3).

10. Determine whether the lines L_1: $x = 1 + 7t$, $y = 3 + t$, $z = 5 - 3t$ and L_2: $x = 4 - t$, $y = 4$, $z = 7 + 2t$ are parallel, intersecting or skew. If they intersect, find the point of intersection.

11. Find equations of all planes containing the x–axis.

12. Find equations of all planes containing the y–axis.

13. Find the angle between the lines $\frac{x-2}{1} = \frac{1-y}{3} = \frac{z-3}{1}$ and $\frac{x}{2} = \frac{y+3}{-1} = \frac{z-1}{2}$.

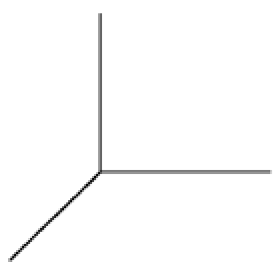
14. Find an equation of the plane containing $P(-1, 2, 1)$ and the line $\frac{x+1}{2} = \frac{y}{5} = \frac{z-3}{1}$.

15. Do the two lines

$$\mathrm{x}_1(t) = \langle 1,1,3\rangle + t\langle -1,0,2\rangle \qquad \text{and} \qquad \mathrm{x}_2 = \langle -1,1,4\rangle + s\langle 2,0,1\rangle$$

intersect? If so, find the point of intersection.

16. Sketch the curve of intersection of the two surfaces $z = y^2 - x^2$ and $x^2 + y^2 = 1$.



17. Let $f(x, y) = 5$

(a) Evaluate $f(-1,-1)$.

(b) Find the domain of f.

(c) Find the range of f.

18. If $Q = \left(1, \frac{\pi}{2}, 3\right)$ in cylindrical coordinates, find rectangular coordinates of Q.

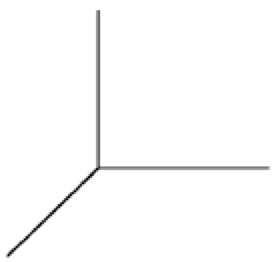
19. Find cylindrical and spherical equations for the surface whose equation in rectangular coordinates is $x = 2$. Describe the surface.

20. Find the set of intersection of the surfaces whose equations in spherical coordinates are $\rho = 4$ and $\theta = \dfrac{\pi}{4}$.

21. Find the set of intersection of the surfaces whose equations in spherical coordinates are $\rho = 4$ and $\theta = \dfrac{\pi}{6}$.

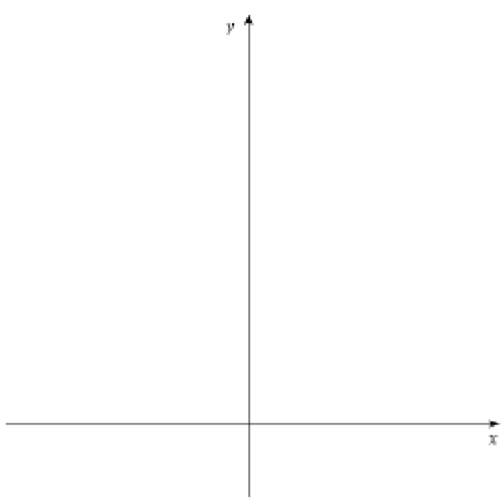
22. Describe in words the solid represented in spherical coordinates by the inequality $2 \le \rho \le 5$.

23. Sketch the solid given in cylindrical coordinates by $0 \le \theta \le \dfrac{\pi}{2}$, $1 \le r \le 3$, $r \le z \le 3$.



24. A curve is given by the vector equation $\mathbf{r}(t) = (2 + \cos t)\,\mathbf{i} + (1 + \sin t)\,\mathbf{j}$. Find a relation between x and y which has the same graph.

25. Consider the curve in the xy-plane defined parametrically by $x = t^3 - 3t$, $y = t^2$, $z = 0$. Sketch a rough graph of the curve.



26. Show that the curve with vector equation $\mathbf{r}(t) = 2\cos^2 t\,\mathbf{i} + \sin(2t)\,\mathbf{j} + 2\sin t\,\mathbf{k}$ is the curve of intersection of the surfaces $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 + z^2 = 4$ Use this fact to sketch the curve.

27. Let $\mathbf{u}(t) = 2t\mathbf{i} + \sin t\,\mathbf{j} - \cos t\,\mathbf{k}$ and $\mathbf{v}(t) = \mathbf{i} + t^2\mathbf{j} - t\mathbf{k}$. Find $\frac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right]$.

28. If $\mathbf{u}(t) = \left\langle -\sqrt{t}\sin t, t, t^{\frac{2}{3}} \right\rangle$ and $\mathbf{v}(t) = \left\langle -\sqrt{t}\sin t, \cos^2 t, -t^{\frac{1}{3}} \right\rangle$, compute $\frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{v}(t)\right)$ and $\frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{u}(t)\right)$.

29. A helix has radius 5 and height 6, and makes 4 revolutions. Find parametric equations of this helix. What is the arc length of the helix?

30. Find the center of the osculating circle of the parabola $y = x^2$ at the origin.

31. Find the center of the osculating circle of the curve described by $x = 4\sin t$, $y = 3t$, $z = 4\cos t$ at $(0,0,4)$.

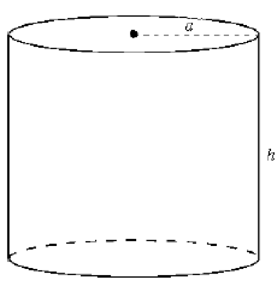
32. Consider $y = \sin x, -\pi < x < \pi$. Determine graphically where the curvature is maximal and minimal.

33. Find a parametric representation for the surface consisting of that part of the elliptic paraboloid $x + y^2 + 2z^2 = 4$ that lies in front of the plane $x = 0$.

34. Find a parametric representation for the surface consisting of that part of the hyperboloid $-x^2 - y^2 + z^2 = 1$ that lies below the disk $\left\{(x,y)\,\middle|\,x^2 + y^2 \le 4\right\}$.

35. Are the two planes $\mathbf{r}_1(s,t) = \langle 1+s+t, s-t, 1+2s\rangle$ and $\mathbf{r}_2(s,t) = \langle 2+s+2t, 3+t, s+3t\rangle$ parallel? Justify your answer.

36. A picture of a circular cylinder with radius a and height h is given below. Find a parametric representation of the cylinder.



37. For the function $z = \sqrt{x^2 + y^2 - 1}$, sketch the level curves $z = k$ for $k = 0, 1, 2,$ and 3.

38. Describe the difference between the horizontal trace in $z = k$ for the function $z = f(x,y)$ and the contour curve $f(x,y) = k$

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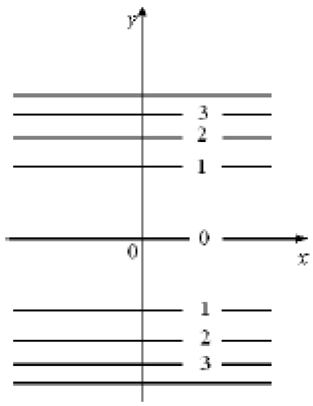
39. The graph of level curves of $f(x,y)$ is given. Find a possible formula for $f(x,y)$ and sketch the surface $z = f(x,y)$.



40. Let $f(x,y) = \begin{cases} \dfrac{x^4 + y^4}{(x^2+y^2)^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ Does this function have a limit at the origin? If so, prove it. If not, demonstrate why not.

41. Determine if $f(x,y) = \begin{cases} \dfrac{x^2 + xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$ is everywhere continuous, and if not, locate the point(s) of discontinuity.

42. Let $f(x,y,z) = (xyz)^2$. Find all second-order partial derivatives of f.

43. If $z = x^2 \sin y + ye^x$, find $\dfrac{\delta z}{\delta x}, \dfrac{\delta z}{\delta y}, \dfrac{\delta^2 z}{\delta x^2}, \dfrac{\delta^2 z}{\delta x \delta y}$, and $\dfrac{\delta^2 z}{\delta y^2}$

44. Find $\dfrac{\delta z}{\delta x}$ for $x^3 - y^2 z + \sin(xyz) = 0$

45. Let $f(x,y,z) = (x-y)(y-z)(z-x)$. Compute $\dfrac{\delta f}{\delta x} + \dfrac{\delta f}{\delta y} + \dfrac{\delta f}{\delta z}$.

46. Show that $f(x,y) = e^y \cos x - 2xy$ satisfies Laplace Equation $f_{xx} + f_{yy} = 0$.

47. If $g(x,y) = \begin{cases} \dfrac{x^4+y^2}{x^2+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ find $\dfrac{\partial g}{\partial x}(1,0)$ and $\dfrac{\partial g}{\partial y}(1,0)$.

48. If $f(x,y) = 4 - x^2$, find $f_x(1,1)$ and $f_y(1,1)$ and interpret these numbers as slopes. Illustrate with sketches.

49. Consider a function of three variables $P = f(A,r,N)$, where P is the monthly mortgage payment in dollars, A is the amount borrowed in dollars, r is the annual interest rate, and N is the number of years before the mortgage is paid off.

(a) Suppose $f(180{,}000, 6, 30) = 1080$. What does this tell you in financial terms?

(b) Suppose $\dfrac{\delta f}{\delta r}(180{,}000, 6, 30) = 115.73$. What does this tell you in financial terms?

(c) Suppose $\dfrac{\delta f}{\delta N}(180{,}000, 6, 30) = -12.86$. What does this tell you in financial terms?

50. If $f(x,y) = ye^{xy}$, find the values x_0 for which $f(x_0, 5) = 5$, and then find an equation of the plane tangent to the graph of f at $(x_0, 5, 5)$.

51. The dimensions of a closed rectangular box are measured to be 60 cm, 40 cm, and 30 cm. The ruler that is used has a possible error in measurement of at most 0.1 cm. Use differentials to estimate the maximum error in the calculated volume of the box.

52. Suppose that $z = x^3 y^2$, where both x and y are changing with time. At a certain instant when $x = 1$ and $y = 2$, x is decreasing at the rate of 2 units/s and y is increasing at the rate of 3 units/s. How fast is z changing at this instant? Is z increasing or decreasing?

53. Suppose that $z = u^2 + uv + v^3$, and that $u = 2x^2 + 3xy$ and $v = 2x - 3y + 2$. Find $\dfrac{\delta z}{\delta x}$ at $(x,y) = (1,2)$.

54. Let $z = \ln\sqrt{x^2+y^2}$, $x = r\cos\theta$, and $y = r\sin\theta$. Use the chain rule to find $\dfrac{\delta z}{\delta r}$ and $\dfrac{\delta z}{\delta \theta}$.

55. Let $w = x^3 + y^3 + z^3$, $x = s + t$, $y = s^2 - t^2$ and $z = st$. Use the chain rule to show that $s(\dfrac{\delta w}{\delta s}) + t(\dfrac{\delta w}{\delta t}) = 3x^3 + 6y^3 + 6z^3$.

56. Let $z = x^2 y + xy^2$ and $x = 2u + v$, and $y = u - 3v$. Show that $\dfrac{\delta^2 z}{\delta u\, \delta v} = -16x - 6y$.

57. Let $f(x,y,z) = x^2y + y^3z + xz^3$ and let $P(2,1,-1)$.

    (a) Find the directional derivative at P in the direction of $\langle 1,2,3\rangle$.

    (b) In what direction does f increase most rapidly?

    (c) What is the maximum rate of change of f at the point P?

58. Find the directional derivative of $f(x,y,z) = x^2 + y^2 - z$ at the point $(1,3,5)$ in the direction of $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

59. Let $f(x,y,z) = x^2 + y^2 + xz$. Find the directional derivative of f at $(1,2,0)$ in the direction of the vector $\mathbf{v} = \langle 1,-1,1\rangle$.

60. Let the temperature in a flat plate be given by the function $T(x,y) = 3x^2 + 2xy$. What is the value of the directional derivative of this function at the point $(3.-6)$ in the direction $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$? In what direction is the plate cooling most rapidly at $(3,-6)$?

61. Suppose that the equation $F(x,y,z) = 0$ defines z implicitly as a function of x and y. Let (a,b,c) be a point such that $F(a,b,c) = 0$ and $\nabla F(a,b,c) = \langle 2,-3,4\rangle$. Find $\dfrac{\delta z}{\delta x}(a,b)$ and $\dfrac{\delta z}{\delta y}(a,b)$.

62. Find $\displaystyle\lim_{h \to 0} \frac{\ln((x+h)^2y) - \ln(x^2y)}{h}$

63. Find an equation of the tangent plane to the surface $4x^2 - y^2 + 3z^2 = 10$ at the point $(2,-3,1)$.

64. Find an equation of the tangent plane to the surface $z = f(x,y) = x^3y^4$ at the point $(-1,2,-16)$.

65. Find the points on the hyperboloid of one sheet $x^2 + y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $2x - y + z = 3$.

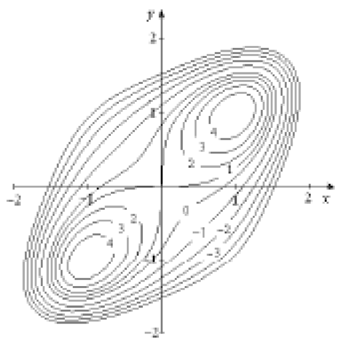
66. Let $f(x,y) = x^2e^y$ and $P(-2,0)$.

    (a) Find the rate of change in the direction of $\nabla f(P)$.

    (b) Calculate $D_{\mathbf{u}}f(P)$ where $\mathbf{u}$ is a unit vector making an angle $\theta = 135°$ with $\nabla f(P)$.

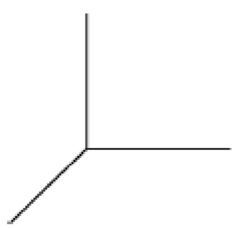
67. Find the local maximum and minimum values and saddle points of the function $f(x,y) = x^3 - 3xy - y^3$.

68. Use the level curves of $f(x,y)$ shown below to estimate the critical points of f. Indicate whether f has a saddle point or a local maximum or minimum at each of those points.



69. Find the critical points (if any) for $f(x,y) = \dfrac{1}{x} + \dfrac{1}{y} + xy$ and determine if each is a local extreme value or a saddle point.

70. Compare the minimum value of z and sketch a portion of the graph of $z = 3x^2 + 6x + 2y^2 - 8y$ near its lowest point.



71. Find the absolute maximum and minimum value of $f(x,y) = x^2 - 3y^2 - 2x + 6y$ on the square region D with vertices $(0,0)$, $(0,2)$, $(2,2)$, and $(2,0)$.

72. Give an example of a non-constant function $f(x, y)$ such that the average value of f over $R = \{(x,y) \mid -1 \le x \le 1, -1 \le y \le 1\}$ is 0.

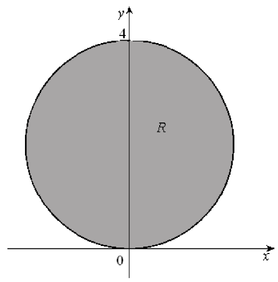
73. Compute $\int_0^1 \int_0^1 xy^2 e^{xy^3}\, dx\, dy$..

74. Evaluate the iterated integral $\int_1^2 \int_0^{\pi/x} x^2 \sin xy \; dy\, dx$.

75. Use a double integral to find the volume of the solid bounded by the planes $x + 4y + 3z = 12$, $x = 0$, $y = 0$, and $z = 0$.

76. Find the volume of the solid under the surface $z = x^2 + y^2$ and lying above the region $\{(x,y) \mid 0 \le x \le 1, x^2 \le y \le \sqrt{x}\}$.

77. Write $\iint_R f(x,y)\,dA$ as an iterated integral in polar coordinates, where R is the region shown below.



78. Find the mass and center of mass of the lamina that occupies the region $D = \{(x,y) \mid 0 \le x \le 2, 0 \le y \le 3\}$ and has density function $p(x,y) = y$.

79. Find the moments of inertia I_x, I_y, and I_0 for the lamina that occupies the region given by $D = \{(x,y) \mid 0 \le x \le 2, 0 \le y \le 3\}$.

80. Find the y-coordinate of the centroid of the semiannular plane region given by $1 \le x^2 + y^2 \le 4$, $y \ge 0$. Sketch the plane region and plot the centroid in the graph.

81. Compute the area of that part of the graph of $3z = 5 + 2x^{3/2} + 4y^{3/2}$ which lies above the rectangular region in the first quadrant of the xy-plane bounded by the lines $x = 0$, $x = 3$, $y = 0$, and $y = 6$.

82. Find the area of that part of the plane $2x + 3y - z + 1 = 0$ that lies above the rectangle $[1, 4] \times [2, 4]$.

83. Find the area of that part of the surface $z = x + y^2$ that lies above the triangle with vertices (0, 0), (1, 1), and (0, 1).

84. Find the area of the surface with vector equation $\mathbf{r}(s, t) = \langle s\cos t, s\sin t, s\rangle$, $1 \le s \le 5$, $0 \le t \le 2\pi$.

85. Find the area of the surface with vector equation $\mathbf{r}(u, v) = \langle u\cos v, u\sin v, u^2\rangle$, $0 \le u \le 2$, $0 \le v \le 2\pi$.

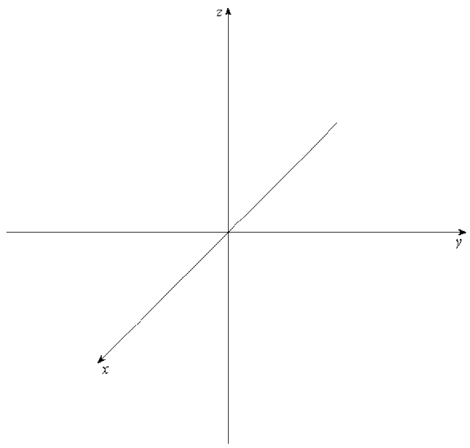
86. Find the volume, using triple integrals, of the region in the first octant beneath the plane $x + 2y + 3z = 6$.

87. Use the method of iterated integration in order to evaluate the triple integral $\iiint_N x\,dV$ where N is the region cut o from the first octant by the plane defined by $x + y + z = 3$.

88. Evaluate $\iiint_E e^{(x^2+y^2+z^2)^{3/2}}\, dV$, where E is the solid bounded by the sphere $x^2+y^2+z^2=1$.

89. A sphere of radius k has a volume of $\frac{4}{3}\pi k^3$. Set up the iterated integrals in rectangular, cylindrical, and spherical coordinates to show this.

90. Sketch the region E whose volume is given by the integral $\int_0^{2\pi}\int_{\pi/6}^{5\pi/6}\int_{1/\sin\phi}^{2} \rho^2 \sin\phi \, d\rho \; d\phi \; d\theta$.



91. Give a geometric description of the solid S whose volume in spherical coordinates is given by
$V=\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2 \sin\phi \, d\rho \; d\phi \; d\theta$.

92. Use the change of variables $x=au$, $y=bv$, $z=cw$ to evaluate $\iiint_E y\, dV$, where E is the solid enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

93. Compute the Jacobian of the transformation T given by $x=\frac{1}{\sqrt{2}}(u-v)$, $y=\frac{1}{\sqrt{2}}(u+v)$, and find the image of $S=\{(u,v) \mid 0\le u\le 1, 0\le v\le 1\}$ under T.

94. Evaluate $\iint_R (x+y)e^{x^2-y^2}\, dA$, where R is the rectangular region bounded by the lines $x+y=0$, $x+y=1$, $x-y=0$, and $x-y=1$.

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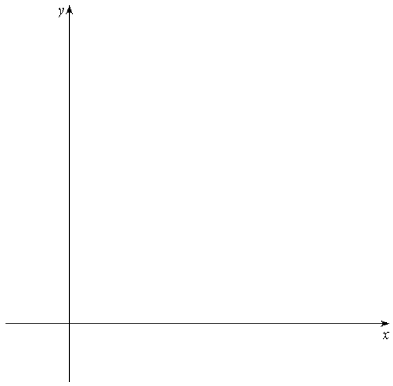
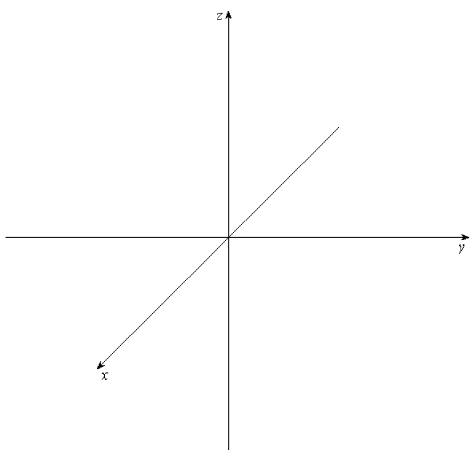
95. Sketch the vector field $\mathbf{F}$ where $\mathbf{F}\left(x,y\right) = x\mathbf{i} - y\mathbf{j}$.



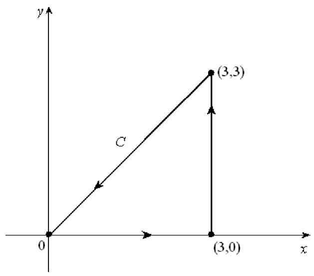
96. Sketch the vector field $\mathbf{F}$ where $\mathbf{F}\left(x,y,z\right) = z\mathbf{k}$.



97. Determine the points $\left(x,y,z\right)$ where the gradient field $\nabla f\left(x,y,z\right)$ for $f\left(x,y,z\right) = xy + xz + yz$ has z-component 0.

98. Find a function of $f\left(x,y\right)$ such that $\nabla f = \mathbf{F}\left(x,y\right) = \left\langle \dfrac{x}{\left(x^2+y^2\right)^{3/2}}, \dfrac{y}{\left(x^2+y^2\right)^{3/2}} \right\rangle$..

99. Evaluate the line integral $\int_C x\, dy$, where the curve C is given in the figure below.



100. Determine whether $\mathbf{F}(x, y, z) = \left\langle 2xy + \ln z,\ x^2 + z\cos(yz),\ \frac{x}{z} + y\cos(yz) \right\rangle$ is conservative and if so, find a potential function.

101. Determine whether $\mathbf{F}(x, y, z) = \langle x - y, y, z \rangle$ is conservative and if so, find a potential function.

102. Evaluate $\int_c \left(e^x + \cos x + y\right)dx + \left(\frac{1}{1+y^2} + ye^{y^2} + x\right)dy$ if C is the path starting at $(0,0)$ and going along the line segment from $(0,0)$ to $(1,1)$, and then along the line segment from $(1,1)$ to $(2,0)$.

103. Evaluate $\int_C \left[3x^2y^2 + 2\cos(2x+y)\right]dx + \left[2x^3y + \cos(2x+y)\right]dy$ if C is the closed path starting at $(0,0)$ and moving clockwise around the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$.

104. Let C be the closed path from $(0,0)$ to $(2,4)$ along $y = x^2$ and back again from $(2,4)$ to $(0,0)$ along $y = 2x$. Evaluate $\int_C \left(x^3 + 2y\right)dx + \left(x - y^2\right)dy$ directly, and then using Green's Theorem.

105. Use Green's Theorem to evaluate the line integral along the given positively oriented curve: $\int_C \left(y^2 - \tan^{-1}x\right)dx + \left(3x + \sin y\right)dy$, where C is the boundary of the region enclosed by the parabola $y = x^2$ and the line $y = 4$.

106. Use Green's Theorem to find the area of the region formed by the intersection of $x^2 + y^2 \le 2$ and $y \ge 1$.

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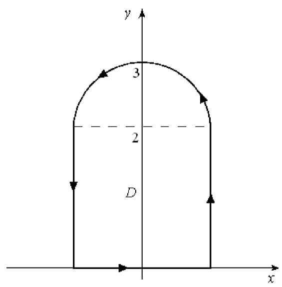
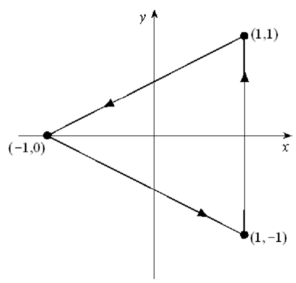
107. Evaluate $\oint_C \left(\frac{1}{4}y^4 - \frac{1}{2}x^3\cos y^3\right)dx + \left(xy^3 + x^2 + \frac{3}{8}x^4y^2\sin y^3\right)dy$, if C is the path shown below.



108. Evaluate $\oint_C 0\,dx + 4x\,dy$, if C is the path shown below, starting and ending at $(1,1)$.

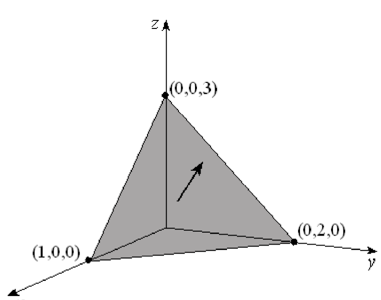


109. Find (a) the curl and (b) the divergence of the vector field $\mathbf{F}(x, y, z) = x^2\mathbf{y}\mathbf{i} + yz^2\mathbf{j} + zx^2\mathbf{k}$.

110. Find (a) the curl and (b) the divergence of the vector field $\mathbf{F}(x, y, z) = \sin x\mathbf{i} + \cos x\mathbf{j} + z^2\mathbf{k}$.

111. Evaluate the surface integral $\int\int_S \mathbf{F}\cdot d\mathbf{S}$ for the vector field $\mathbf{F}(x, y, z) = x^2y\mathbf{i} - 3xy^2\mathbf{j} + 4y^3\mathbf{k}$ where S is the part of the elliptic paraboloid $z = x^2 + y^2 - 9$ that lies below the square $0 \le x \le 1,\ 0 \le y \le 1$ and has downward orientation.

112. Evaluate the flux of the vector field $\mathbf{F} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ through the plane region with the given orientation as shown below.



113. Find the flux of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ across the paraboloid given by $x = u\cos v,\ y = u\sin v,\ z = 1 - u^2$ with $1 \le u \le 2,\ 0 \le v \le 2\pi$ and upward orientation:

114. Use Stokes' Theorem to evaluate $\int_C xy\,dx + yz\,dy + zx\,dz$, where C is the triangle with vertices $(1,0,0), (0,1,0)$, and $(0,0,1)$, oriented counterclockwise as viewed from above.

115. Evaluate the flux integral $\iint_S (2x\mathbf{i} - y\mathbf{j} + 3z\mathbf{k}) \cdot \mathbf{n}\,dS$ over the boundary of the ball $x^2 + y^2 + z^2 \le 9$.

116. Let $\mathbf{F}(x, y, z) = x^2 y\,\mathbf{i} - x^2 z\mathbf{j} + z^2 y\mathbf{k}$ and let S be the surface of the rectangular box bounded by the planes $x = 0,\ x = 3,\ y = 0,\ y = 2,\ z = 0$, and $z = 1$. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathrm{S}$.

117. Let $\mathbf{F}(x, y, z) = \dfrac{z\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}}{\left(x^2 + y^2 + z^2\right)^{3/2}}$ and let S be the boundary surface of the solid $E = \left\{(x, y, z) \,\middle|\, 1 \le x^2 + y^2 + z^2 \le 4\right\}$. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

118. Find the flux of $\mathbf{F}(x, y, z) = x^3\mathbf{i} + 2xz^2\mathbf{j} + 3y^2 z\,\mathbf{k}$ across the surface of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy-plane.

119. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \dfrac{x}{\left(x^2 + y^2 + z^2\right)^{3/2}}\mathbf{i} + \dfrac{y}{\left(x^2 + y^2 + z^2\right)^{3/2}}\mathbf{j} + \dfrac{z}{\left(x^2 + y^2 + z^2\right)^{3/2}}\mathbf{k}$ and S is the sphere $x^2 + y^2 + z^2 = 9$.

Final Exam Review
Answer Section

SHORT ANSWER

1. ANS:
The closest distance between the two spheres is 1:5. (Hint: The centers of two spheres are $(1,-1,0)$ and $(3,-2,-2)$ and distance between centers is 3. Sketch the two spheres.)

PTS: 1

2. ANS:
$|\mathbf{a}|=\sqrt{14}$, $\mathbf{a}+\mathbf{b}=\langle 3,8,6\rangle$, $\mathbf{a}-\mathbf{b}=\langle 3,-4,-8\rangle$, $2\mathbf{a}=\langle 6,4,-2\rangle$, and $3\mathbf{a}+4\mathbf{b}=\langle 9,30,25\rangle$

PTS: 1

3. ANS:
$s=3,\ t=-\dfrac{1}{2}$

PTS: 1

4. ANS:
(i) $-\sqrt{3}$, (ii) $4-3\sqrt{3}$, (iii) -3, (iv) $8+4\sqrt{3}$

PTS: 1

5. ANS:
$\mathrm{proj}_{\mathbf{b}}\,\mathbf{a}=\langle -1,2,2\rangle$

PTS: 1

6. ANS:
$\langle -6,4,6\rangle$

PTS: 1

7. ANS:
91

PTS: 1

8. ANS:
21

PTS: 1

9. ANS:
$$\frac{x-2}{2}=\frac{y+3}{1}=\frac{z-4}{2}$$

PTS: 1

10. ANS:
Skew

PTS: 1

11. ANS:
$by + cz = 0$ where b and c are not both zero.

PTS: 1

12. ANS:
$ax + cz = 0$ where a and c are not both zero.

PTS: 1

13. ANS:
$\cos^{-1} \dfrac{7}{3\sqrt{11}} \approx 45.3°$ or 0.7904 radians

PTS: 1

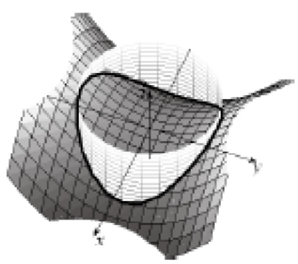
14. ANS:
$3x - y - z + 6 = 0$

PTS: 1

15. ANS:
Yes; $\left(\dfrac{1}{5}, 1, \dfrac{23}{5}\right)$

PTS: 1

16. ANS:



PTS: 1

17. ANS:
(a) 5
(b) x and y can be any real numbers.
(c) $\{5\}$

PTS: 1

18. ANS:
(0, 1, 3)

PTS: 1

19. ANS:
Cylindrical: $r\cos\theta = 2$; spherical: $\rho\sin\phi\cos\theta = 2$

PTS: 1

20. ANS:
A half circle radius 4 on the plane $y = x$

PTS: 1

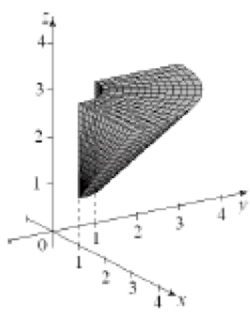
21. ANS:
A circle radius 4 on the plane $z = 2\sqrt{3}$

PTS: 1

22. ANS:
solid sphere of radius 5 centered at origin, with hollow ball inside of radius 2

PTS: 1

23. ANS:

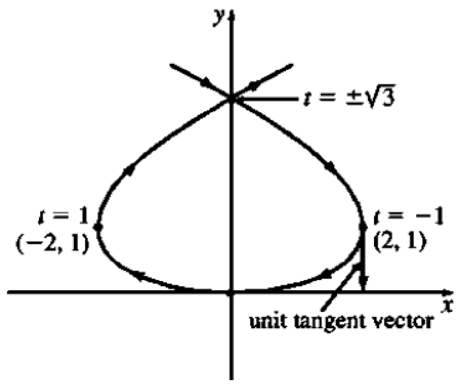


PTS: 1

24. ANS:
$(x - 2)^2 + (y - 1)^2 = 1$

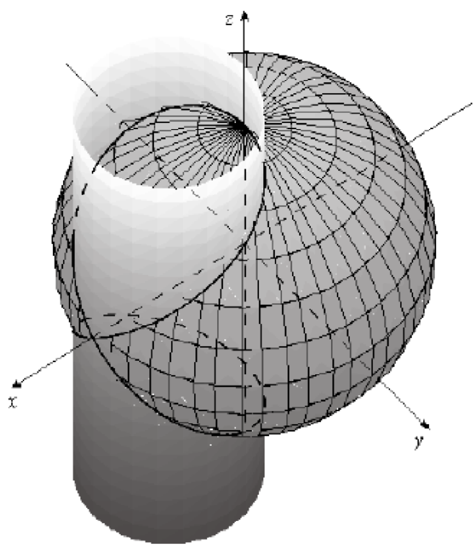
PTS: 1

25. ANS:



PTS: 1

26. ANS:



PTS: 1

27. ANS:

$$\frac{d}{dt}\left[\mathbf{u}(t)\times\mathbf{v}(t)\right]=\left(t\cos t-\sin t-t^2\sin t\right)\mathbf{i}+(4t+\sin t)\mathbf{j}+\left(6t^2-\cos t\right)\mathbf{k}$$

PTS: 1

28. ANS:

$$\frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{v}(t)\right)=0,\ \frac{d}{dt}\left(\mathbf{u}(t)\cdot\mathbf{u}(t)\right)=\sin^2 t+2t\sin t\cos t+2t+\frac{4}{3}t^{\frac{1}{3}}$$

PTS: 1

29. ANS:
$\mathbf{r}(t) = \left\langle 5\cos 2\pi t, 5\sin 2\pi t, \frac{3}{2}t \right\rangle, 0 \le t \le 4$; $L = \sqrt{36 + (40\pi)^2} \approx 125.8$

PTS: 1

30. ANS:
$\left(0, \frac{1}{2}\right)$

PTS: 1

31. ANS:
$(0, 0, -2.25)$

PTS: 1

32. ANS:
Minimal at $x = 0$, maximal at $x = \pm\frac{\pi}{2}$

PTS: 1

33. ANS:
$x = 4 - y^2 - 2z^2$, $y = y$, $z = z$ where $y^2 + 2z^2 \le 4$ since $x \ge 0$.

PTS: 1

34. ANS:
$x = r\cos\theta$, $y = r\sin\theta$, $z = -\sqrt{1 + r^2}$, $0 \le \theta \le 2\pi$, $0 \le r \le 2$

PTS: 1

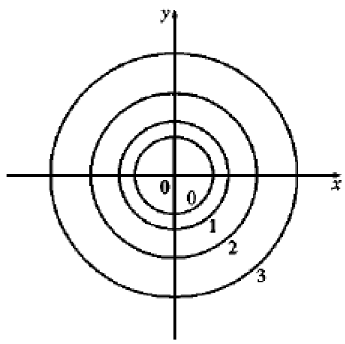
35. ANS:
Yes

PTS: 1

36. ANS:
$\mathbf{r}(s, t) = \langle a\cos s, a\sin s, t\rangle$, $0 \le s \le 2\pi$, $0 \le t \le h$. Other answers are possible.

PTS: 1

37. ANS:

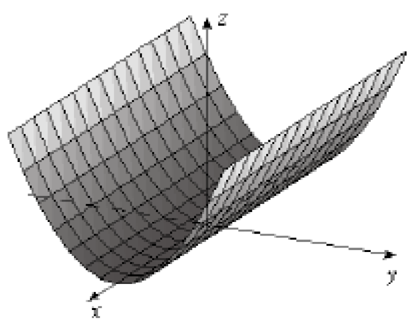


PTS: 1

38. ANS:
The contour curve $f(x,y) = k$ is the horizontal trace of $z = f(x,y)$ in $z = k$ projected down to the xy-plane.

PTS: 1

39. ANS:
$z = y^2$. There are other possible answers.



PTS: 1

40. ANS:
Approaching the origin along $y = 0$, the limit equals 1; approaching the origin along $y = x$, the limit equals $\frac{1}{2}$. Thus, this function does not have a limit at the origin.

PTS: 1

41. ANS:
f is continuous everywhere its denominator does not equal zero. The limit does not exist at (0,0) and thus f is discontinuous at (0,0).

PTS: 1

42. ANS:
$f_{xy} = 4xyz^2$, $f_{yx} = 4xyz^2$, $f_{zx} = 4xy^2z$, $f_{xz} = 4xy^2z$, $f_{yz} = 4x^2yz$, $f_{zy} = 4x^2yz$, $f_{xx} = 2y^2z^2$, $f_{yy} = 2x^2z^2$, $f_{zz} = 2x^2y^2$

PTS: 1

43. ANS:
$\frac{\delta z}{\delta x} = 2x\sin y + ye^x, \frac{\delta z}{\delta y} = x^2\cos y + e^x, \frac{\delta^2 z}{\delta x^2} = 2\sin y + ye^x, \frac{\delta^2 z}{\delta x \delta y} = 2x\cos y + e^x$, and $\frac{\delta^2 z}{\delta y^2} = -x^2 \sin y$

PTS: 1

44. ANS:
$$\frac{\delta z}{\delta x} = \frac{-3x^2 - yz\cos(xyz)}{-y^2 + xy\cos(xyz)}$$

PTS: 1

45. ANS:
$$\frac{\delta f}{\delta x} = (y-z)(z-x) - (x-y)(y-z)$$

$$\frac{\delta f}{\delta y} = -(y-z)(z-x) + (x-y)(z-x)$$

$$\frac{\delta f}{\delta z} = (x-y)(y-z) - (x-y)(z-x)$$

$$\frac{\delta f}{\delta x} + \frac{\delta f}{\delta y} + \frac{\delta f}{\delta z} = 0$$

PTS: 1

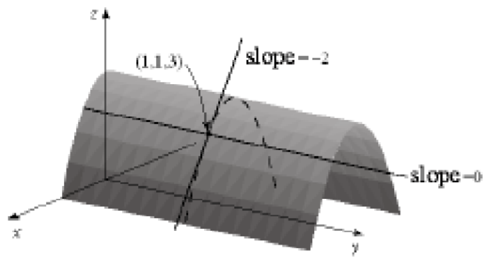
46. ANS:
$f_x = -e^y \sin x - 2y$, $f_y = e^y \cos x - 2x \Rightarrow f_{xx} = -e^y \cos x, f_{yy} = e^y \cos x \Rightarrow f_{xx} + f_{yy} = 0$

PTS: 1

47. ANS:
$\frac{\delta g}{\delta x}(1,0) = 2$, $\frac{\delta g}{\delta y}(1,0) = 0$

PTS: 1

48. ANS:
$f_x(1,1) = -2,\ f_y(1,1) = 0$



PTS: 1

49. ANS:
(a) If you borrow $180,000 with annual interest rate 6% and 30 year amortization, then your monthly payment is $1080.

(b) If you borrow $180,000 with annual interest rate 6% and 30 year amortization, then your monthly payment increases by $115.73 per 1% increase in interest rate.

(c) If you borrow $180,000 with annual interest rate 6% and 30 year amortization, then your monthly payment decreases by $12.86 for each additional year of amortization.

PTS: 1

50. ANS:
$x_0 = 0; z = 25x + y$

PTS: 1

51. ANS:
540 cm^3

PTS: 1

52. ANS:
$\dfrac{dz}{dt} = -12$, z is decreasing

PTS: 1

53. ANS:
180

PTS: 1

54. ANS:
$$\frac{\delta z}{\delta r} = \frac{x\cos\theta}{x^2+y^2} + \frac{y\sin\theta}{x^2+y^2} = \frac{1}{r},\ \frac{\delta z}{\delta\theta} = \frac{-x\sin\theta}{\sqrt{x^2+y^2}} + \frac{y\cos\theta}{\sqrt{x^2+y^2}} = 0.$$

PTS: 1

55. ANS:
Answers may vary

PTS: 1

56. ANS:
Answers may vary

PTS: 1

57. ANS:
(a) $\dfrac{26}{\sqrt{14}}$

(b) $\langle 3,1,7\rangle$

(c) $\sqrt{59}$

PTS: 1

58. ANS:
$\dfrac{-2\sqrt{21}}{7}$

PTS: 1

59. ANS:
$\dfrac{-1}{\sqrt{3}}$

PTS: 1

60. ANS:
$\dfrac{6}{5}$; the plate is cooling most rapidly in the direction $-\mathbf{i}-\mathbf{j}$

PTS: 1

61. ANS:
$\dfrac{\delta z}{\delta x}(a,b) = -\dfrac{1}{2}, \ \dfrac{\delta z}{\delta y}(a,b) = \dfrac{3}{4}$

PTS: 1

62. ANS:
$\dfrac{2}{x}$

PTS: 1

63. ANS:
$16x + 6y + 6z = 20$

PTS: 1

64. ANS:
$48x - 32y - z + 96 = 0$

PTS: 1

65. ANS:
$\left(1, -\frac{1}{2}, -\frac{1}{2}\right), \left(-1, \frac{1}{2}, \frac{1}{2}\right)$

PTS: 1

66. ANS:
(a) $4\sqrt{2}$

(b) $D_{\mathbf{u}} f(P) = |\nabla f(P)| \cos 135° = 4\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -4$

PTS: 1

67. ANS:
$(0,0)$ is a saddle point; $f(-1,1) = 1$ is a local maximum

PTS: 1

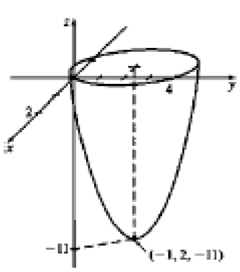
68. ANS:
f has a saddle point at $(0,0)$ and maximum points at $(-1,-1)$ and $(1,1)$.

PTS: 1

69. ANS:
$f(1,1) = 3$ is a local minimum.

PTS: 1

70. ANS:
$z(-1,2) = -11$ is the absolute minimum value.



PTS: 1

71. ANS:
Absolute maximum value is 2, and absolute minimum is 0.

PTS: 1

72. ANS:
Possible functions include $f(x, y) = x$ and $f(x, y) = y$. Any linear function of x and y with no constant term will work, as will many other functions.

PTS: 1

73. ANS:
$\frac{1}{3}(e-2)$

PTS: 1

74. ANS:
3

PTS: 1

75. ANS:
24

PTS: 1

76. ANS:
$\frac{6}{35}$

PTS: 1

77. ANS:
$\int_0^{\pi}\int_0^{4\sin\theta} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$

PTS: 1

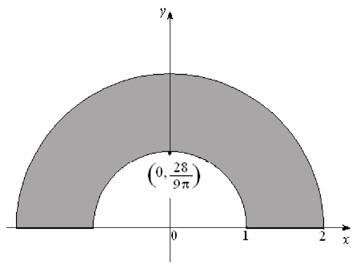
78. ANS:
$m = 9$, $(\bar{x}, \bar{y}) = (1, 2)$

PTS: 1

79. ANS:
$I_x = 18, I_y = 8, I_0 = 26$

PTS: 1

80. ANS:
$\left(0, \frac{28}{9\pi}\right)$



PTS: 1

81. ANS:
$\frac{4}{15}(392\sqrt{7} - 789)$

PTS: 1

82. ANS:
$6\sqrt{14}$

PTS: 1

83. ANS:
$\frac{1}{2}\sqrt{6} - \frac{1}{6}\sqrt{2}$

PTS: 1

84. ANS:
$24\sqrt{2}\pi$

PTS: 1

85. ANS:
$\frac{\pi(17\sqrt{17} - 1)}{6}$

PTS: 1

86. ANS:
6

PTS: 1

87. ANS:
$\frac{27}{8}$

PTS: 1

88. ANS:
$$\frac{4\pi(e-1)}{3}$$

PTS: 1

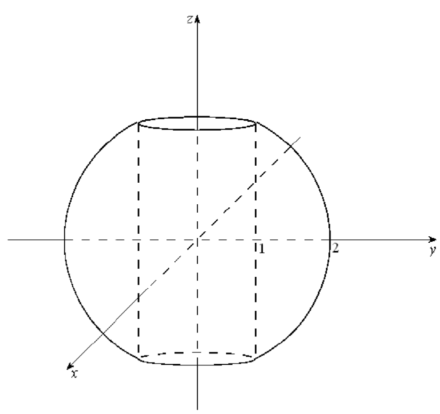
89. ANS:

Rectangular coordinates: $V=\int_{-k}^{k}\int_{-\sqrt{k^2-x^2}}^{\sqrt{k^2-x^2}}\int_{-\sqrt{k^2-x^2-y^2}}^{\sqrt{k^2-x^2-y^2}} dz\,dy\,dx$; cylindrical coordinates:

$V=\int_{0}^{2\pi}\int_{0}^{k}\int_{-\sqrt{k^2-r^2}}^{\sqrt{k^2-r^2}} r\,dz\,dr\,d\theta$; spherical coordinates: $V=\int_{0}^{2\pi}\int_{0}^{\pi}\int_{0}^{k}\rho^2\sin\phi\,d\rho\;d\phi\;d\theta$

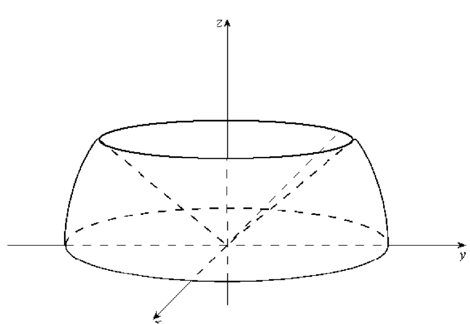
PTS: 1

90. ANS:
A sphere of radius 2 with a hole of radius 1 drilled through the center.



PTS: 1

91. ANS:
A hemisphere with a cone cut out.



PTS: 1

92. ANS:
0

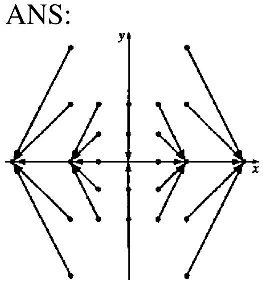
PTS: 1

93. ANS:
$J = 1$, image of $S = \{(x,y) \mid -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}, |x| \le y \le \sqrt{2} - |x|\}$

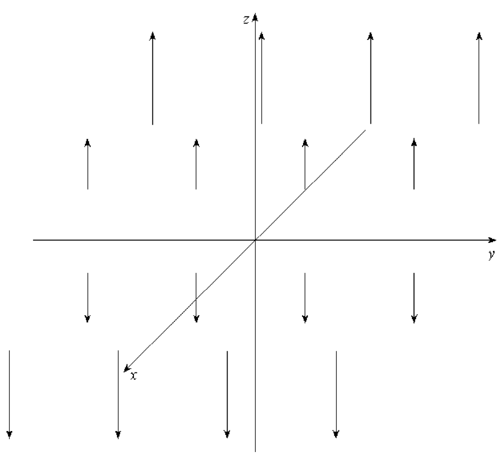
PTS: 1

94. ANS:
$\frac{e-2}{2}$

PTS: 1

95. ANS:


PTS: 1

96. ANS:


PTS: 1

97. ANS:
All points of the form $(a,-a,b)$

PTS: 1

98. ANS:
$$f\left(x,y\right)=\frac{-1}{\left(x^2+y^2\right)^{1/2}}+C$$

PTS: 1

99. ANS:
$\frac{9}{2}$

PTS: 1

100. ANS:
$f(x, y, z) = x^2 y + \sin(yz) + x \ln z$

PTS: 1

101. ANS:
F is not conservative.

PTS: 1

102. ANS:
$e^2 + \sin 2 - 1$

PTS: 1

103. ANS:
0

PTS: 1

104. ANS:
$-\frac{4}{3}$

PTS: 1

105. ANS:
$-\frac{96}{5}$

PTS: 1

106. ANS:
$\frac{\pi}{2} - 1$

PTS: 1

107. ANS:
0

PTS: 1

108. ANS:
8

PTS: 1

109. ANS:
(a) $-\left(2yz\mathbf{i}+2xz\mathbf{j}+x^2\mathbf{k}\right)$
(b) $2xy+z^2+x^2$

PTS: 1

110. ANS:
(a) $-\sin x\,\mathbf{k}$
(b) $\cos\ x+2z$

PTS: 1

111. ANS:
$-\frac{3}{2}$

PTS: 1

112. ANS:
$\frac{21}{2}$

PTS: 1

113. ANS:
18π

PTS: 1

114. ANS:
$-\frac{1}{2}$

PTS: 1

115. ANS:
144π

PTS: 1

116. ANS:
24

PTS: 1

117. ANS:
0

PTS: 1

118. ANS:
32π

PTS: 1

119. ANS:
4π

PTS: 1