MATH 2400 EXAM 1 REVIEW

Disclaimer: By no means is this review to be considered complete. You are responsible for all materials from class, written and online homework, and the book.

(1) Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 6x + 4y - 2z = 8.$$

(2) Let $\vec{a} = 4\vec{i} + \vec{j}$ and $\vec{b} = \vec{i} - 2\vec{j} + 3\vec{k}$ be 3-dimensional vectors. Find
  (a) $2\vec{a} - \vec{b}$
  (b) $\|\vec{a}\|$
  (c) $\vec{a} \cdot \vec{b}$
  (d) The angle between $\vec{a}$ and $\vec{b}$

(3) Find the work done by a force $\vec{F} = \vec{i} - 6\vec{j} + 2\vec{k}$ that moves an object from the point $(0, 10, 8)$ to the point $(1, 6, 12)$ along a straight line. The distance is measured in meters and the force in Newtons.

(4) Let $P(3, -2, 0), Q(4, 0, 1)$, and $R(1, 2, 1)$ be points in $\mathbb{R}^3$. Find
  (a) The distance from P to Q
  (b) The area of the triangle with vertices P, Q and R
  (c) The equation of the plane containing P, Q and R

(5) Let $\vec{a}, \vec{b}$, and $\vec{c}$ be 3-dimensional vectors. Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}.$$

(6) Let $z = f(x, y) = 2 - \sqrt{x^2 + y^2}$.
  (a) Find the domain and range of f.
  (b) Draw the horizontal traces of f for $z = 1$ and $z = -1$.
  (c) Sketch a graph of f.
  (d) Write the equation for f in cylindrical coordinates.

(7) Find parametric equations for the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t, y = 1 - t, z = 2t$.

(8) Which quadric surfaces do the following equations represent?
  (a) $x^2 + \frac{y^2}{7} + z^2 = 1$
  (b) $x^2 + \frac{y^2}{7} - z^2 = 1$
  (c) $z = x^2 - \frac{y^2}{7}$
  (d) $z^2 = x^2 + \frac{y^2}{7}$

(9) Change the following points from spherical to rectangular coordinates.
   (a) $(2, \pi, \frac{\pi}{2})$
   (b) $(3, 0, \frac{3\pi}{2})$

(10) Identify the surfaces whose equations are given.
   (a) $r = 3$
   (b) $\theta = \frac{\pi}{3}$
   (c) $\phi = \frac{\pi}{3}$