MATH 2400: Calculus III, Fall 2013
MIDTERM #1

September 18, 2013

YOUR NAME:

Circle Your CORRECT Section

001 E. Angel ..... (9am)
002 E. Angel ..... (10am)
003 A. Nita ..... (11am)
004 K. Selker ..... (12pm)
005 I. Mishev ..... (1pm)
006 C. Farsi ..... (2pm)
007 R. Rosenbaum ..... (3pm)
008 S. Henry ..... (9am)

Important note: SHOW ALL WORK. BOX YOUR ANSWERS. Calculators are not allowed. No books, notes, etc. Throughout this exam, please provide exact answers where possible. That is: if the answer is 1/2, do not write 0.499 or something of that sort; if the answer is π, do not write 3.14159 or something of that sort. **Put your name on each page. BREATHE!!! Good luck.**

problem	points	score
1	12	
2	10	
3	12	
4	18	
5	20	
6	12	
7	16	
TOTAL	100 pts	

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

SIGNATURE:

NAME:	SECTION:

1. (??? points) A plane, heading due North at an airspeed of $1,000$ km/hr, experiences a wind of 100 km/hr blowing towards North East.

(a) Find the plane's ground speed vector.

(b) Find the magnitute of the plane's ground speed vector.

(c) Compute the angle that determines the direction of the plane's motion vector, that is the angle between the ground speed vector and the unit due North vector.

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2. (??? points)

(a) Find a function $f(x, y, z)$ whose level surface at 2 is the cylinder of radius 1 centered on the x-axis.

(b) Find a function $g(x, y, z)$ whose level surface at 2 is a paraboloid centered on the y-axis which contains the point $(0, 1, 0)$ and opens in the positive y direction.

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3. (12 points) Let $\vec{u} = s\vec{j} + \vec{k}$ and $\vec{v} = \vec{i} - 2\vec{j} + t\vec{k}$.

(a) Compute $\vec{u} \times \vec{v}$.

(b) Are there any values of s and t that will make $\vec{u}$ and $\vec{v}$ parallel? Why or why not?

(c) If the magnitude of $\vec{u}$ is $\sqrt{2}$, for what number pair(s) are $\vec{u}$ and $\vec{v}$ perpendicular to one another?

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4. (18 points) Let T be the triangle with vertices at $A(1,2,3)$, $B(3,3,6)$, and $C(2,2,5)$. Let $\vec{N} = 6\vec{i} - 3\vec{j} - 3\vec{k}$.

(a) Show that $\vec{N}$ and $\overrightarrow{AB} \times \overrightarrow{AC}$ are parallel to one another.

(b) Find an equation of the form $ax + by + cz = d$ for the plane that T lies in.

(c) Find the area of T.

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5. (20 points) Let $\vec{u} = \langle 3, 2, 7 \rangle$ and $\vec{v} = \langle -1, 5, 2 \rangle$.

(a) Decompose $\vec{u}$ into component vectors $\vec{u}_{\parallel}$ and $u_{\perp}$ which are, respectively, parallel and perpendicular to $\vec{v}$, and verify that $\vec{u} = \vec{u}_{\parallel} + \vec{u}_{\perp}$.

(b) If $\vec{w} = \langle 1, 0, 1 \rangle$, find the volume of the parallelepiped three of whose sides are $\vec{u}$, $\vec{v}$, and $\vec{w}$.

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6. (12 points) Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}, \quad (x, y) \mapsto y^2 - x + 10$.

(a) Find an equation for the contour of f that goes through $(4, -1)$.

(b) Sketch a contour diagram of f with at least 3 level curves. Include the level curve found in (a). Label each curve with the value that f has on it.

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7. (16 points) Determine for each of the following functions whether it is continuous at the given point. In each case, justify your conclusion carefully.

(a) The function

$$g\colon \mathbb{R}^2 \to \mathbb{R},\ (x,y) \mapsto g(x,y) = \begin{cases} \frac{x^4}{(x^2+y^2)^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$$

at the point $(0,0)$.

(b) The function

$$h\colon \mathbb{R}^2 \to \mathbb{R},\ (x,y) \mapsto h(x,y) = \begin{cases} \frac{e^{x^2+y^2}-1}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 1 & \text{if } (x,y) = (0,0), \end{cases}$$

at the point $(0,0)$.