MATH 2400: Calculus III, Fall 2013
Final Exam

December 16, 2013

NAME AND SIGNATURE:

"On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work."

Circle your section.

001 E. Angel .... (9am)

002 E. Angel .... (10am)

003 A. Nita .... (11am)

004 K. Selker .... (12pm)

005 I. Mishev .... (1pm)

006 C. Farsi .... (2pm)

007 R. Rosenbaum .... (3pm)

008 S. Henry .... (9am)

You must show all of your work. Please write legibly and box your answers. The use of calculators, books, notes, etc. is not permitted on this exam. Please provide exact answers when possible. For example, if the answer is π, write the symbol "π" and not the decimal 3.14159. . . .

Question	Points	Score
1	20	
2	30	
3	30	
4	40	
5	40	
6	40	
Total:	200	

1. (20 points) The functions and vector fields below are all defined on the whole 3–space and they are assumed to be smooth on their domain.

   (a) Which of the following integrals are always zero? Circle all that apply. Note that you will lose points if you circle an integral that is not always zero.

   (i) $\int_C \operatorname{grad} f \cdot d\vec{r}$ for all smooth functions f and all closed curves C in 3–space.

   (ii) $\iint_S \operatorname{grad} f \cdot d\vec{A}$ for all smooth functions f and all closed surfaces S in 3–space.

   (iii) $\iint_S \operatorname{curl} \vec{F} \cdot d\vec{A}$ for all smooth vector fields $\vec{F}$ and all closed surfaces S in 3–space.

   (iv) $\iiint_W \operatorname{div} \vec{F}\, dV$ for all smooth vector fields $\vec{F}$ and all three-dimensional regions W in 3–space.

   (b) Assume that $\operatorname{curl} \vec{F} = 0$ on the whole 3–space. What can we say about $\int_C \vec{F} \cdot d\vec{r}$? (here C is a closed curve in 3–space.)

   (c) Assume that $\operatorname{div} \vec{F} = 0$ on the whole 3–space. What can we say about $\iint_S \vec{F} \cdot d\vec{A}$? (here S is a closed surface in 3–space.)

2. (30 points) Find and classify all the critical points of $f(x, y) = x^3 + 3x^2 + 3y^2 - 12x + 32$.

3. (30 points) Evaluate the line integral

$$\int_C (-z\vec{i} + y\vec{j} + x\vec{k}) \cdot d\vec{r}$$

where C is the circle of radius 3 in the plane $x + 2y + 2z = 5$ centered at the point $(5, 0, 0)$, oriented counterclockwise when viewed from the origin (*Hint*: Use Stokes' theorem).

4. (40 points) (a) Find the flux of $\vec{F} = x\vec{i} + y\vec{j}$ through the surface S, where S is the part of the paraboloid $z = 1 + x^2 + y^2, 1 \leq z \leq 5$, oriented upward.

   (b) Find the flux of $\vec{F} = 2x\vec{i} - 3\vec{j} + z\vec{k}$ out of the sphere S of radius 2 centered at the origin (*Hint*: Use the Divergence theorem).

5. (40 points) Determine for each of the following functions whether it is continuous at the given point. In each case, justify your conclusion carefully.

   (a) The function $g\colon \mathbb{R}^2 \to \mathbb{R}$ given by

   $$g(x,y) = \begin{cases} \dfrac{x^4}{(x^2+y^2)^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   at the point $(0,0)$.

   (b) The function $h\colon \mathbb{R}^2 \to \mathbb{R}$ given by

   $$h(x,y) = \begin{cases} \dfrac{e^{x^2+y^2}-1}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

   at the point $(0,0)$.

6. (40 points) In this problem you will evaluate the integral

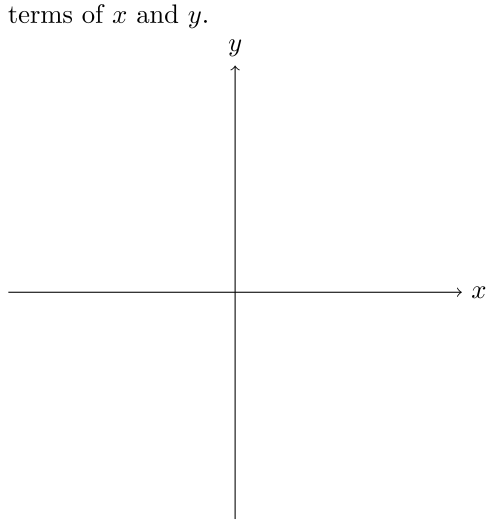
$$I = \iint_D (x+y)e^{x^2-y^2}\, dA,$$

where D is the diamond with corners $(0,0)$, $(1,1)$, $(2,0)$ and $(1,-1)$. Let

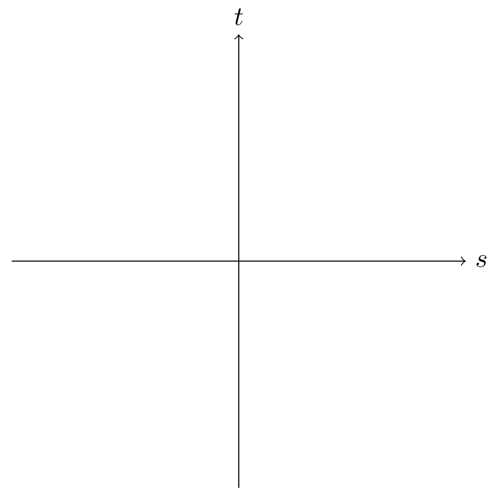
$$s = x + y \quad \text{and} \quad t = x - y.$$

(a) Calculate the Jacobian $\dfrac{\partial(x,y)}{\partial(s,t)}$.

(b) Draw a picture of the diamond D on the provided graph. Find equations for the four edges of D in terms of x and y.



(c) Convert the equations from (b) into equations involving only s and t. Draw the region bounded by these equations in the st-plane on the provided graph.



(d) Write the integral I in terms of s and t, and evaluate it. (*Hint*: One order of integration may be easier than the other. You may also find a u-substitution helpful.)