1. (15) Write the Cartesian equation of the following surfaces and describe their graphs:

(i) $z^2 - 2r^2 = 4$;

(ii) $\rho = 8\cos\phi$.

2. (20) Consider the function

$$f(x,y) \;=\; \frac{2xy^2}{x^2+y^2}, \quad (x,y) \neq (0,0).$$

Compute

$$\lim_{(x,y)\to(0,0)} f(x,y),$$

and define $f(0,0)$ in way that extends f to be continuous at the origin.

3. (15) Consider the function f defined on the set $\{(x, y) \mid x > 0,\ y > 0\}$ given by

$$f(x, y) = e^x + x\ln y + y\ln x.$$

(i) Show that $f_{xy} = f_{yx}$ on the domain $\{(x, y) \mid x > 0,\ y > 0\}$ of f.

(ii) Write an equation for the tangent plane $\mathcal{P}$ to the graph of f at the point $(1, 1, e)$.

4. (20) Find the critical points of the function $f(x, y) = -x^3 + 4xy - 2y^2 + 1$ and classify the critical points using the second derivative test.

5. (15) Use linear approximation to approximate $(\sqrt{24} + (28)^{1/3})^2$. (You may leave your answer as a fraction.)

6. (10) Assume z is a function of x and y, (i.e., $z = z(x, y)$). If $x^5 + xz^3 + yz = 3$, find $\dfrac{\partial z}{\partial x}$ at the point $(1, 1, 1)$.

7. (5) Find the largest possible domain of the function

$$\frac{(1 + \sin xy)e^{-x^2-y^2}}{\sqrt{x^2 + y^2}}.$$

Name: _____

Section: _____

University of Colorado

Mathematics 2400: Second Midterm Exam

October 5, 2005

Problem	Points	Score
1	15	
2	20	
3	15	
4	20	
5	15	
6	10	
7	5	
Total	100	