MIDTERM 1 REVIEW

Disclaimer: By no means is this review to be considered complete. You are responsible for all materials from class, written and online homework, and the book.

(1) Suppose $\vec{a}, \vec{b}$ and $\vec{c}$ are vectors in $\mathbb{R}^3$ such that $\vec{a}+\vec{b}+\vec{c}=0$. Show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$.

(2) Describe geometrically with words and sketch the solutions to the following equations
  (a) $x=5$ in $\mathbb{R}^1$, $\mathbb{R}^2$ and $\mathbb{R}^3$
  (b) $(x-1)^2+y^2=36$ in $\mathbb{R}^2$ and $\mathbb{R}^3$
  (c) $z=y^2$ in $\mathbb{R}^3$
  (d) $y=8z-3x+1$ in $\mathbb{R}^3$

(3) Using the concept of vectors, find the distance from the point $(-2,3)$ to the line $3x-4y+5=0$ in $\mathbb{R}^2$. The distance from a point P to a line L is defined to be length of the shortest line which connects P to the line L. Hint: Draw a picture of the shortest line. What do you notice about it?

(4) Sketch contour diagrams of the following functions.
  (a) $z=f(x,y)=8x-2y$
  (b) $z=f(x,y)=\frac{x-y}{y}$

(5) Suppose $z=f(x,y)$. Can level curves for different z-values overlap (i.e. contain the same point)? Does (4b) contradict this?

(6) Determine which of the following functions are continuous at the given points. Be sure to justify your answer.
  (a) The function

$$f:\mathbb{R}^2\to\mathbb{R}, (x,y)\mapsto f(x,y)=\begin{cases}\dfrac{xy^3}{x^2+y^6} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{if } (x,y)=(0,0)\end{cases}$$

  at the point $(0,0)$.
  (b) The function

$$g:\mathbb{R}^2\to\mathbb{R}, (x,y)\mapsto g(x,y)=\left(x^2+y^2\right)^{3/2}$$

  at the point $(0,0)$.
  (c) The function

$$h:\mathbb{R}^2\to\mathbb{R}, (x,y)\mapsto h(x,y)=x+5y$$

  at the point $(1,2)$.

(7) Let S be the sphere of radius 4 centered at $(1,1,\sqrt{3})$.
  (a) Write down an equation for S.
  (b) Verify that $P=(1+\sqrt{3},2,3\sqrt{3})$ lies on S.
  (c) Determine the plane tangent to the sphere at the point P. Hint: How do vectors from the center of the sphere to a point on the sphere relate to the tangent plane at that point?

(8) Let $\vec{u} = \langle 1, 0, 1, -5\rangle$ and $\vec{v} = \langle -2, 101, 7, 1\rangle$ be vectors in $\mathbb{R}^4$. Are they parallel? Are they perpendicular? Can you use the cross product to produce a vector perpendicular to both of them?

(9) Find the angle between a diagonal of a cube and one of its edges.

(10) Consider the plane $z = 3x - y + 11$.

  (a) Find a unit normal vector to the plane pointing downward. Call this vector $\vec{u}$.

  (b) Verify that $P = (0, 0, 11), Q = (0, 1, 10)$ and $R = (1, 0, 14)$ are points on the plane.

  (c) Compute the cross product $\overrightarrow{PQ} \times \overrightarrow{PR}$. Call this vector $\vec{v}$.

  (d) Compute the cross product $\vec{u} \times \vec{v}$. Explain your answer geometrically.

(11) Find the area of the (simple) quadrilateral with vertices $(1, 1)$, $(6, 3)$, $(7, 6)$ and $(2, 4)$ using concepts from class.