Even delta-matroids and the complexity of planar Boolean CSPs

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History

- Our world: CSP$(\{0,1\},\Gamma)$ where Γ contains constants $\{0\}$ and $\{1\}$.
- We limit the instance shape – each variable appears at most k times. For which Γs do we get easier CSP?
- T. Feder: Fanout limitations on constraint systems, 2001.
- V. Dalmau, D. Ford: Generalized satisfiability with k occurences per variable: A study through delta-matroid parity, 2003.
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Edge CSP

- Wlog each variable appears in exactly two constrains.
- We can draw instances of this CSP as graphs with variables = edges.
- Some people call this binary CSP, we prefer edge CSP.
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Δ-matroids

- AKA "generalized matroids"
- $R \neq \emptyset$ is an even Δ-matroid if all tuples in R have the same parity and for all $\alpha, \beta \in R$ and for all u variables such that $\alpha(u) \neq \beta(u)$ there exists $v \neq u$ such that $\alpha(v) \neq \beta(v)$ and $\alpha \oplus u \oplus v \in R$:
- Δ-matroids: No parity restriction, enought to have $\alpha \oplus u \in R$ instead of $\alpha \oplus u \oplus v \in R$.

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Good and bad news about Δ-matroids

- Intersection of two even Δ-matroids need not be a Δ-matroid:

$$\begin{Bmatrix} (0 & 0 & 0 & 0) \\ (1 & 1 & 0 & 0) \\ (0 & 0 & 1 & 1) \\ (1 & 1 & 1 & 1) \end{Bmatrix} \cap \begin{Bmatrix} (0 & 0 & 0 & 0) \\ (1 & 0 & 1 & 0) \\ (0 & 1 & 0 & 1) \\ (1 & 1 & 1 & 1) \end{Bmatrix} = \begin{Bmatrix} (0 & 0 & 0 & 0) \\ (1 & 1 & 1 & 1) \end{Bmatrix}$$

- If there is any way to use polymorphisms here, we did not find it.
- However, (even) Δ-matroids are closed under primitive positive definitions where each bound variable appears exactly twice and each free variable exactly once.

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Edge CSPs generalize perfect matchings

- Perfect matchings in graphs: Given G, assign 0 or 1 to each edge so that each vertex of G is incident to exactly one edge labelled by 1.
- Known to be polynomial (J. Edmonds, 1965).
- Perfect matchings correspond to edge CSP with constraints of the form

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Planar CSPs

- Z. Dvořák and M. Kupec: On Planar Boolean CSP, 2015.
- CSP($\{0, 1\}, \Gamma$) with incidence graphs of instances planar.
- Constraints – faces of a planar graph, variables – vertices.
- Dvořák and Kupec show that all interesting cases of planar CSP can be reduced to edge CSP with Δ-matroid constraints.
- If there is a polynomial algorithm for edge CSP with even Δ-matroid constraints, we have a dichotomy for planar CSP.

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Our strategy

- We generalize Edmond's blossom algorithm for perfect matchings.
- Edge labeling f assigns 0 or 1 to each half-edge: Pair $\{v, C\}$ where v lies in constraint C so that all constraints are satisfied.
- Variable is consistent in f if both half edges corresponding to v have the same labels.
- Edge labeling with all variables consistent = a solution of the instance.
- We want to augment a given labeling f: Find g labeling with fewer inconsistencies.
- If f is an edge labeling that can be improved, there is an augmenting f-walk p from one inconsistent variable to another.

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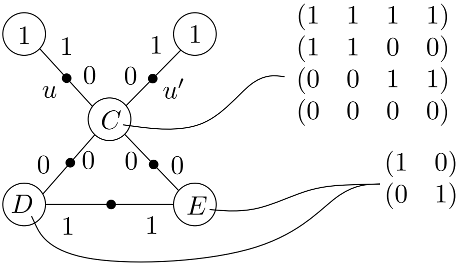
Example



Sketch of the algorithm

- Take f, search from all inconsistent variables, building a forest of visited variables and constraints.
- If we can find f-walks $u \dots Cv$ and $u' \dots Dv$ for u, u' inconsistent, we can augment and make u, u' consistent.
- If we find f-walks $u \dots Cv$ and $u \dots Dv$, we have found a blossom. This we contract and re-run the algorithm on a smaller instance.
- If we don't find any of the above, then f can not be augmented.

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Consequences and future work

- We have finished the classification started by Dvořák and Kupec.
- To get dichotomy for edge CSPs, all that is needed is to generalize our argument from even Δ-matroids to all Δ-matroids.
- We can go beyond even Δ-matroids and cover many previously known polynomial classes, but there still remains a large gap.
- We are now begining to look at valued version of edge CSP for even Δ-matroids.
- Generalization to value sets larger than 2 is going to be hard.

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