**WORK PROBLEMS - pumping CALCULUS 2 NAME\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

Recall from yesterday: W = F \* d

Same idea – but now we’re considering the amount done in pumping a liquid from one location to another.

We will need to visualize the situation again – a picture. We also need to determine how we are determining a small amount of work.

**Example**: An upright cylindrical tank is 5 ft tall, radius of 2 ft and is 80% full of water, which has a density of 62.4 lb per cubic ft. How much work is done in pumping all the water to the top of the tank.

For this problem – sketch the picture. NOW – imagine the pumping – we do NOT pump all the water at once. RATHER – think of taking one very small slice of the water and lifting it to the top of the tank.

If the tank is 80% full, the top of the water level is at the 4 ft mark. Let’s consider how much work would be done in moving a very small slice taken at this point to the top of the tank:

W = F \* d

F = weight of the water = vol of the water \* density

??What does one “slice” of the water appear to be?? A very thin cylinder – it’s volume is:

 (WHY?? Talk to your neighbor to make sure everyone understands!!)

Therefore: F = 

 (Notice what happened to the units -- what unit is Force now expressed in???\_\_\_\_\_\_\_\_\_\_\_)

To determine the work, we now multiply Force and distance. We have represented the Force – what about the distance?

At x = 4: How far does the “slice” move? 1 ft

  (Do the units look correct for “work”?)

At x = 3: How far does a “slice” move? 2 ft

 

At x = 2: How far does a “slice” move? 3 ft

 

Can we generalize for any value of x? YES!



Now to determine ALL the work done in pumping all the water, we need to “accumulate” the work of moving ALL the slices. **(INTEGRATE)**



QUESTION: What did “dx” represent in this problem? What was the physical attribute associated with it?

2nd QUESTION: What if we wanted to label the top of the water as x=0 and the bottom of the tank as x=4?

**How does that change the problem?????**

The DISTANCE each layer moves changes:

At x = 0, d = 1 ft

At x = 1, d = 2 ft

At x=2, d = 3 ft

In general: at x, d = x+1 ft *-- this changes our integral – but will it change out answer???*

 

Example problems: In each problem, sketch a picture & determine how is Force & distance represented in each. ***Use this to create the expression for work!***

**1.** A gas stationstores its gasoline in a tank under the ground. The tank is a cylinder standing in upright position. If the radius of the cylinder is 4 *ft*, its height is 12 *ft*, and the top is 15 *ft* under the ground, find the total work needed to pump the gasoline out the tank. Gasoline weighs 42 lb per cubic foot. The tank is full of gasoline. Do the same problem when the tank is half full. ***(What changes?)***

**2.** A water tank is in shape of right circular cone with the height 30 *ft* and radius 8 *ft* at the top. Find the work required to pump all of it over the top of the tank. Recall that 1 cubic foot of water weighs 62.4 *lb*

A. If the tank is filled with water to a depth of 20 *ft*..

B. If the tank is full. ***(What changes?)***

C. If the tank is full and the water is pumped to a height of 10 *ft* above the top of the tank. ***(What changes?)***

**3.** A gas stationstores its gasoline in a tank under the ground. The tank is a cylinder lying horizontally on its side. If the radius of the cylinder is 5 *ft*, its length is 15 *ft*, and the top is 12 *ft* under the ground, find the total work needed to pump the gasoline out the tank. Gasoline weighs 42 lb per cubic foot. The tank is full of gasoline.