Calculating Volumes (slices)

VISUALIZE THE SOLID.

1. Draw a careful graph of the region.  
Label the curves.  
Decide what parts of the curves are the actual *boundaries* for the region.  
Locate intersection points by solving equations simultaneously.

2. Make a 3-dimensional sketch of the solid.

PICTURE A SLICE.

3. Decide which way to slice.  
See which variable (*x* or *y*) directly specifies the location of any slice. This will be your variable of integration.  
Picture the slices corresponding to various values of this variable.  
Think about how the slices vary.

4. Show a typical slice on your drawing of the region. This is an edge-on view.  
Draw a corresponding face-on view.

EXPRESS THE AREA OF EACH SLICE.

5. Notice that the area of any slice *depends* on the variable of integration (*x* or *y*).  
You may need to go back and forth between your drawing of the region and your face-on view of the slice.  
Express the area of the typical slice *in terms of* your variable of integration.

SET UP AND EVALUATE THE INTEGRAL.

6. The volume of each slab is simply the area of the slice times the thickness (*dx* or *dy*).  
Express the volume of the solid as a definite integral.  
The limits of the integral are the boundary values of the variable of integration.

7. Calculate the volume by evaluating the definite integral.

**EXAMPLE**

The base of a solid is formed by the area enclosed in the xy-plane by the function and y = 4.



1. Sketch a picture of the solid formed if cross-sections are squares that are perpendicular to the x-axis.
2. Represent the volume of 1 slice of the solid
3. Set up the integral that would be utilized to calculate the volume of the entire solid.
4. Find the volume of the solid.

The base of the solid is formed by the same functions described above

1. Sketch a picture of the solid formed if cross-sections are semi-circles that are perpendicular to the x-axis.
2. Represent the volume of 1 slice of the solid
3. Set up the integral that would be utilized to calculate the volume of the entire solid.
4. Find the volume of the solid.

The base of the solid is formed by the same functions described above

1. Sketch a picture of the solid formed if cross-sections are squares that are perpendicular to the y-axis.
2. Represent the volume of 1 slice of the solid
3. Set up the integral that would be utilized to calculate the volume of the entire solid.
4. Find the volume of the solid.