MATH 2300 PROJECT 1: Integration of trigonometric functions

By using trig identities combined with u/du substitution we'd like to evaluate integrals of the form $\int \sin^m x \cos^n x\, dx$ (for integer values of n and m). The goal of this project is for you to work together to discover the techniques that work for these anti-derivatives.

1. Warm-up problem: $\int \cos^4 x \sin x\, dx$

2. $\displaystyle\int \sin^3 x\, dx$ (Hint: Use the identity $\sin^2 x + \cos^2 x = 1$. Then make a substitution.)

3. $\displaystyle\int \sin^5 x \cos^2 x\, dx$ (Hint: Write $\sin^5 x$ as $(\sin^2 x)^2 \sin x$.)

4. $\displaystyle\int \sin^7 x \cos^5 x\, dx$ (The algebra gets hairy - stop once the substitution is complete, you don't need to fully evaluate.)

5. In general, how would you go about trying to find $\displaystyle\int \sin^m x \cos^n x\, dx$, where m is odd? (Hint: consider the previous three problems.)

6. Note that the same kind of trick works when the power on $\cos x$ is odd. To check that you understand, what trig identity and what u/du substitution would you use to integrate $\displaystyle\int \cos^3 x \sin^2 x\, dx$?

7. Now what if the power on $\cos x$ and $\sin x$ are both even? Find $\int \sin^2 x \, dx$, in each of the following two ways:

(a) Use the identity $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$.

(b) Integrate by parts, with $u = \sin x$ and $dv = \sin x \, dx$.

(c) Show that your answers to parts (a) and (b) above are the same. Hint: Use a double angle formula. This problem reminds us of proving trig identities in precalculus!

(d) How would you evaluate the integral $\int \sin^2 x \cos^2 x \, dx$?

8. Do the integral in problem (2), above, again, but this time by parts, using $u = \sin^2 x$ and $dv = \sin x\, dx$. (After this, you'll probably need to do a substitution.)

9. For fun: Can you show your answers to problems (2) and (8) above are the same? It's another great trigonometric identity.

10. Further investigations you may consider (especially for mathematics, physics, and engineering majors): We also would like to be able to solve integrals of the form $\int \tan^m x \sec^n x\, dx$. These two functions play well with each other, since the derivative of $\tan x$ is $\sec^2 x$ and the derivative of $\sec x$ is $\sec x \tan x$, and since there is a pythagorean identity relating them. It sometimes works to use $u = \tan x$ and it sometimes works to use $u = \sec x$. Based on the values of m and n, which substitution should you use? Are there cases for which neither substitution works?