## TAYLOR SERIES MATCHING

**Purpose.** This activity is designed to help develop and reinforce students' understanding of creating Taylor series representations of functions. It also can be used to practice converting between sigma-notation and expanded form of Taylor series.

**Background knowledge.** Before doing this activity, students should know that most of the functions they are familiar with can be represented by Taylor series. In particular, they should know the Taylor series representations (centered about $a = 0$) for $e^x$, $\sin x$, $\cos x$, and $\dfrac{1}{1-x}$. They should have been exposed to the idea that series representations for new functions can be built from known Taylor series using the techniques of substituting, multiplying by a constant or a power of $x$, differentiating, or integrating.

**Preparation (before class) and implementation (in class).** The activity is designed to be done in groups, with three or four students per group.

Before class, copy each page—ideally, each page should be on a different color of paper. The pages can be laminated before cutting up the cards and shuffling. The front page contains 12 function cards, the second page contains 12 Taylor series cards that match the functions, and the last page contains the same 12 series cards, but in expanded form.

Put the students into groups and give them the cards. We have found that you do not need to give any directions at all. Figuring out what to do can be part of the activity, and solving the puzzle of what to do serves as a warm-up for the students. However, if you want to give instructions, you can introduce the project by saying something like:

> "The object is to match each function card (blue) with its Taylor series representation (pink) and with the expanded form of this representation (yellow). You should end with 12 triples, each triple containing a card of each color."

**Alternate uses.** You can omit the function cards, and ask students to match just the sigma-notation form of series with the expanded form of the series. This allows them to practice finding patterns in series, and to develop fluency in converting between sigma-notation and expanded form. This can be done immediately after series and sigma-notation have first been introduced.

You could also have students match the functions with just the sigma-notation form of their representation, or just with the expanded form of their representation.

For a more challenging activity, you could give the students just the set of sigma-notation series cards (or just the set of expanded-form cards), and ask them to produce the functions that they represent.

**Leading questions and general ideas.** As the students explore this activity, certain questions may arise—or you may wish to bring them up to guide the students in their learning:

- Which pairs (or triples) did you match first, and why? Which did you match last, and why?
- What techniques did you use for connecting the series to the Taylor series representations? (substitution, multiplication by a power of $x$, differentiation and integration.)
- When you look at a function, how do you figure out which technique to use to produce the Taylor series representation?
- What algebraic simplification tools were necessary?
- Two of the series start at $n = 1$ rather than $n = 0$. Why?
- Which series looked very similar and yet represent very different functions?
- Can you think of examples of functions that you cannot find the Taylor series representation for by using these methods?
- Create an example of a function/Taylor series that could be added to this activity.

**Debrief.** If possible, leave some time after the activity is completed for questions, and for discussion of the facts, procedures, and ideas that the activity was meant to reinforce.

As you discuss these issues, you might want to write certain facts on the board. For example: make a list of the basic Taylor series representations they should know, and a list of the techniques used in the activity.

**Follow-up challenge.** Time permitting (or to be completed outside of class), you can ask your students to each create an example of a Taylor series for a function, then exchange with their classmates to figure out what function it represents.