

1. Recall the idea of linear approximation, which we will use to approximate the numbers e^{-1} and e . Let $f(x) = e^x$. We want to find a linear function $L(x) = C_0 + C_1x$ whose value and whose derivative at $x = 0$ matches $f(x)$ at $x = 0$.

(a) Fill in the table below:

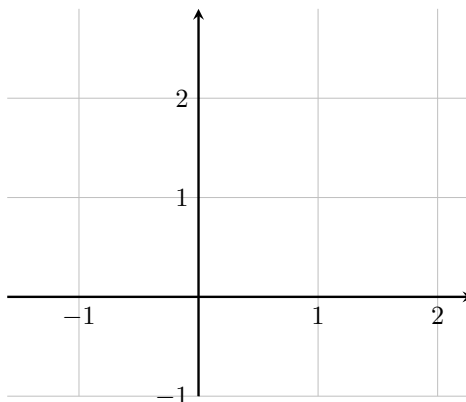
$f(x) =$	$f(0) =$	$L(x) = C_0 + C_1x$	$L(0) = C_0$ (by substituting)
$f'(x) =$	$f'(0) =$	$L'(x) =$	$L'(0) =$

(b) Now we want to match the values and derivatives of $f(x)$ and $L(x)$, in other words:

- $f(0) = L(0)$, so $C_0 =$ _____
- $f'(0) = L'(0)$, so $C_1 =$ _____

(c) Thus $L(x) =$

(d) Graph e^x and $L(x)$



- (e) Use $L(x)$ to estimate $e^{0.1}$ and e . Compare them to the estimates your calculator gives you and comment on the accuracy. Explain from the graph why the error is so much larger for estimating the value of e .

(f) Note: The general formula for a linear approximation centered at $x = 0$ is:

$$L(x) = f(0) + f'(0)x$$

This is the equation of the tangent line, and is also called the 1st degree Taylor Polynomial for $f(x)$ centered at $x = 0$. Confirm that the answer you got in part (c) matches this formula.

2. Now we'll repeat the ideas of the previous problem, but using a quadratic approximation instead of a linear approximation. Let $f(x) = e^x$. We want to find a quadratic function $Q(x) = C_0 + C_1x + C_2x^2$ whose value and whose derivative and whose second derivative at $x = 0$ matches $f(x)$ at $x = 0$.

(a) Fill in the table below:

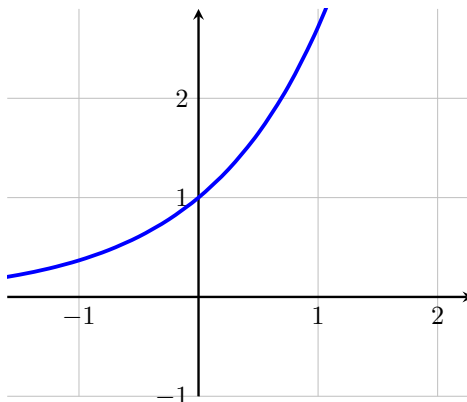
$f(x) =$	$f(0) =$	$Q(x) =$	$Q(0) = C_0$ (by substituting)
$f'(x) =$	$f'(0) =$	$Q'(x) =$	$Q'(0) =$
$f''(x) =$	$f''(0) =$	$Q''(x) =$	$Q''(0) =$

(b) Now we want to match the values and derivatives of $f(x)$ and $Q(x)$, in other words:

- $f(0) = Q(0)$, so $C_0 =$ _____
- $f'(0) = Q'(0)$, so $C_1 =$ _____
- $f''(0) = Q''(0)$, so _____

(c) Thus $Q(x) =$ _____

(d) A graph of e^x is shown below. Graph $Q(x)$ on the same coordinate plane.



- (e) Use $Q(x)$ to estimate $e^{0.1}$ and e . Compare them to the estimates your calculator gives you and comment on the accuracy. Explain from the graph above and the graph from the previous problem why the errors are reduced.

(f) Note: The general formula for a quadratic approximation centered at $x = 0$ is:

$$Q(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

It is also called the 2nd degree Taylor Polynomial for $f(x)$ centered at $x = 0$. Confirm that this is the same as the linear approximation with an additional term added. Confirm that the answer you got in part (c) matches the formula.

3. Now we'll repeat the ideas of the previous problem, but using a cubic approximation. Let $f(x) = e^x$. We want to find a cubic function $T(x) = C_0 + C_1x + C_2x^2 + C_3x^3$ whose value and whose derivative and whose second derivative and whose third derivative at $x = 0$ matches $f(x)$ at $x = 0$.

(a) Fill in the table below:

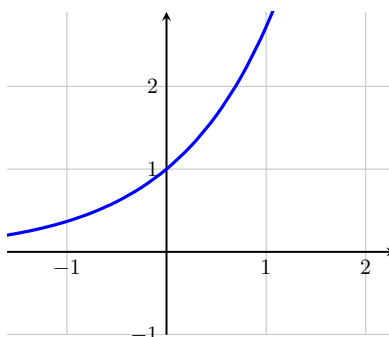
$f(x) =$	$f(0) =$	$T(x) =$	$T(0) =$
$f'(x) =$	$f'(0) =$	$T'(x) =$	$T'(0) =$
$f''(x) =$	$f''(0) =$	$T''(x) =$	$T''(0) =$
$f'''(x) =$	$f'''(0) =$	$T'''(x) =$	$T'''(0) =$

(b) Now we want to match the values and derivatives of $f(x)$ and $T(x)$, in other words:

- $f(0) = T(0)$, so $C_0 =$ _____
- $f'(0) = T'(0)$, so _____
- $f''(0) = T''(0)$, so _____
- $f'''(0) = T'''(0)$, so _____

(c) Thus $T(x) =$ _____

(d) A graph of e^x is shown below. Graph $T(x)$ on the same coordinate plane.



(e) Use $T(x)$ to estimate e^{-1} and e . Compare them to the estimates your calculator gives you and comment on the accuracy. Compare the accuracy to that of the previous two problems.

(f) Note: The general formula for a cubic approximation centered at $x = 0$ is:

$$T(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

This is also called the 3rd degree Taylor Polynomial for $f(x)$ centered at $x = 0$. Confirm that the first three terms are the same as the quadratic approximation, and confirm that your answer to part (c) matches this formula.

4. Fill in the table below, for $f(x) = e^x$. Note that $f^{(n)}(x)$ means the n th derivative of $f(x)$.

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$\frac{f^{(n)}(0)}{n!}$	$\frac{f^{(n)}(0)}{n!}x^n$
0				
1				x
2				
3				
4				
5				
6				

5. Add the terms in the last column to get the 6th degree Taylor Polynomial for $f(x) = e^x$ centered at $x = 0$.
6. Use the Taylor Polynomial from Problem 5 to estimate e .
7. Write down the general formula for the n th degree Taylor polynomial for $f(x)$, centered at $x = 0$.
8. Give an explanation of where the factorials come from.