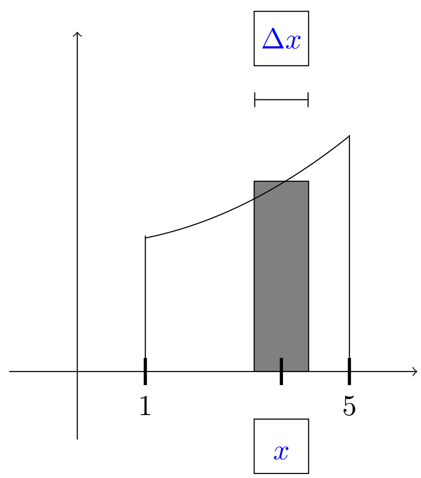
1. Area Underneath a Curve.

   (a) Warmup: The area of a rectangle is height × width. What is the area of a rectangle with height 7 cm and width 2 cm? Include units.

   (b) Consider the region between the curve $f(x) = 1 + x^2$ and the x-axis on the interval $[1, 5]$. The scale on both axes is measured in centimeters. Cut the region under the curve into thin vertical strips. The top of each strip is almost flat, so each strip is approximately a rectangle. Include units.



      i. For the rectangle with base x units from the origin, find its height and width, with units.

      height: ____________________________ width: __________________

      ii. What is the area of this rectangle? Include units.

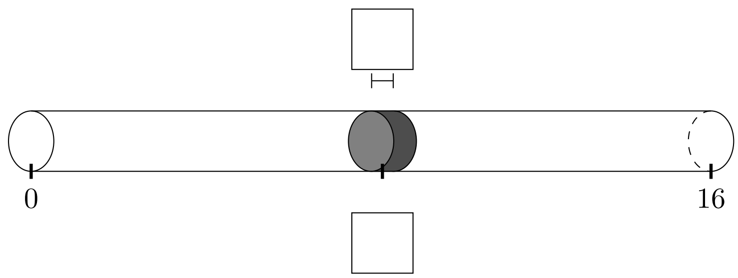
      iii. To find the total area, we "add" the thin rectangles together with an integral. Express the area of the region as a definite integral. Include units.

      iv. Compute the integral you found to determine the area. Include units.

2. Mass of a Rod.

   (a) Warmup: The mass of a thin rod is linear-density $\times$ length. Lead has volume-density of 11.34 g/cm^3. So a lead rod with diameter 1 cm would have linear-density of $\rho = 11.34\ \text{g/cm}^3 \cdot \pi(.5\text{cm})^2 \approx 9$ g/cm. Find the mass of a 8 cm long lead thin rod that has linear-density 9 g/cm. Include units.

   (b) Now suppose a rod with length 16 cm has linear-density $\rho(x) = 1 + \sqrt{x}$ g/cm. Cut the rod into thin slices. When cut thin enough, each slice will have approximately constant density. Fill in the boxes in the diagram below.



      i. For the slice x units from the left, find its density and length, with units.

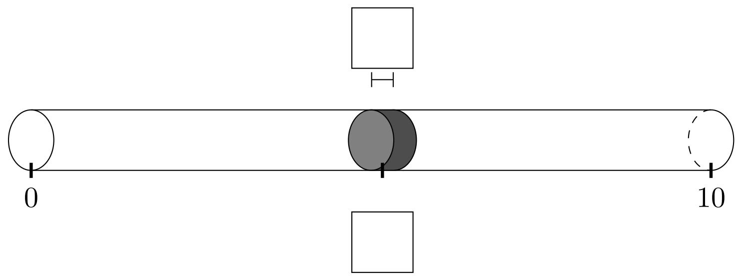
         density: ____________________ length (thickness):________________

      ii.-iv. Find the mass of this thin slice of the rod, then express the mass of the rod as a definite integral, then evaluate the integral to determine the mass. Include units at each step.

3. Energy Consumption.

   (a) Background formula: The energy (measured in kWh = 1000 Watt · hours) needed to run a device with a given power level (in Watts) for a duration of time (in hours) is given by Energy = Power × Time.

   (b) Suppose a house has a non-constant power consumption given by the formula, $P(t) = \frac{170000}{100+t^2}$, with t in hours past noon, and power in Watts. We would like to find out how much energy is needed to power the house from Noon to 10 PM. Divide the 10 hours into very small lengths of time. For small enough stretches of time, the power consumption of the house is approximately constant.



   i.-iv. Using the same steps and thought process as in the previous examples, Find the total energy used. Include units. For comparison, in 2012 the average household in the United States used about 30 kWh of energy per day.

4. Work.

   (a) Background formula: Write down the physics formula for calculating work done. Include SI units.

   (b) Suppose you want to take a 1000 kg satellite from the surface of the Earth and remove it entirely from Earth's gravitational field. The force between two bodies with masses m_1 and m_2 that are a distance r apart is given by the formula

   $$F = G\frac{m_1 m_2}{r^2}$$

   (G is a constant, m_1 and m_2 are the masses, r is the distance between their centers.)

   i.-iv. Following the same procedures and thought processes as in the previous examples, first approximate the work necessary to move the satellite a short distance further from earth. Then express the work done to remove the satellite from Earth's gravitational field using an improper integral. Call the radius of the earth r_e. Compute the improper integral to find the work done. Wait until the end to substitute the values $G \approx 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$; $m_e \approx 5.97 \times 10^{24}$ kg; $r_e \approx 6370$ km.