A beaker contains three types of molecules, called monomers, dimers, and trimers. We use M, D, and T to stand for the quantities of each of the three respective types. Suppose these quantities are changing over time, according to the following "rate equations:"

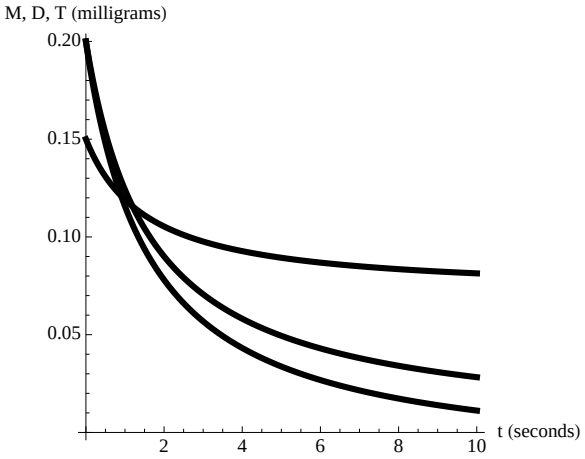
$$\begin{aligned} M' &= -4M^2 - 0.8MD, \\ D' &= 2M^2 - 0.8MD, \\ T' &= 0.8MD. \end{aligned}$$

Let's suppose that, *initially, there are equal* (nonzero) *quantities of monomers and dimers.*

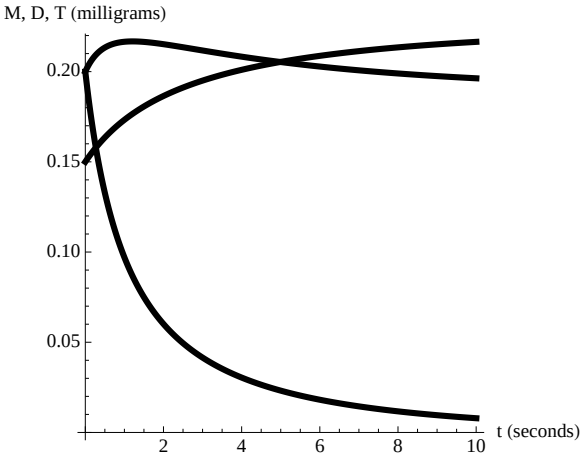
1. Is D initially increasing or decreasing? Please explain.

2. What is the "threshold value" of M/D, meaning the value of the ratio M/D at which D changes from increasing to decreasing (if D is initially increasing), or from decreasing to increasing (if D is initially decreasing)? Please explain.

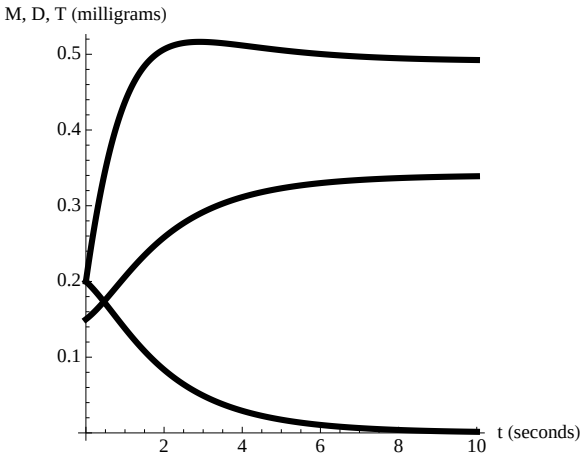
3. Which of the four graphs on the following page could possibly be a graph of the quantities M, D, and T modeled by the above rate equations? Please explain your reasoning carefully, and on the correct graph, label which curve is M, which is D, and which is T. Hint: start by thinking about increase and decrease.


M, D, T (milligrams)
0.20
0.15
0.10
0.05
2
4
6
8
10
t (seconds)

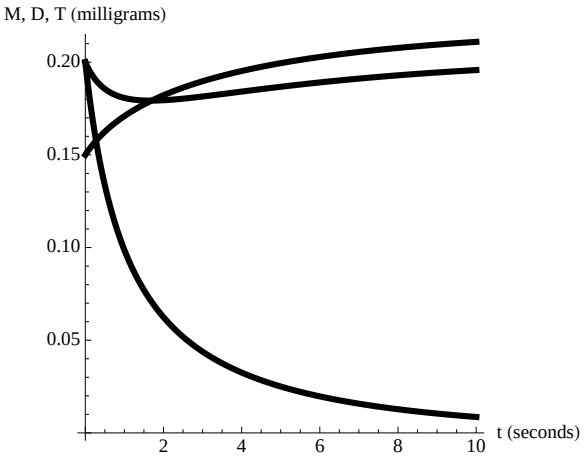
(i)


M, D, T (milligrams)
0.20
0.15
0.10
0.05
2
4
6
8
10
t (seconds)

(ii)


M, D, T (milligrams)
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
t (seconds)

(iii)


M, D, T (milligrams)
0.20
0.15
0.10
0.05
2
4
6
8
10
t (seconds)

(iv)

4. Fill in the blanks (try to answer based primarily on quantitative reasoning and mathematics; you shouldn't need any advanced knowledge of chemical reactions):

   A monomer may react with another monomer to form a dimer. These monomer-to-monomer reactions cause a decrease in the total quantity of _____. Moreover, the rate at which this occurs is proportional to M^2 (since each of the M milligrams of monomers present has roughly M milligrams of other _____ with which to react). The monomer-to-monomer reactions therefore correspond to the term _____ in the above equation for M'.

   Further, whenever two monomers are lost to a monomer-to-monomer reaction, one _____ is gained. That is: the rate at which dimers are gained from such reactions equals half the rate at which_____ are lost to these reactions. Since half of $4M^2$ equals _____, the monomer-to-monomer reactions account for the term _____ in the above equation for D'.

   A monomer may also react with a dimer to form a _____. The rate at which this occurs is proportional to the product of the quantity of monomers and the quantity of dimers (since each of the _____ milligrams of monomers present has _____ milligrams of dimers with which to react). The decrease in M resulting from these monomer-to-dimer reactions therefore corresponds to the term _____ in the above equation for M'. Analogously, the decrease in D resulting from these monomer-to-dimer reactions corresponds to the term _____ in the above equation for D'.

   Finally, when a monomer and a dimer are lost to a monomer-to-dimer reaction, one _____ is gained. This accounts for the term _____ in the above equation for T'.

5. Use the rate equations on the first page, above, to compute $M' + 2D' + 3T'$. What does this tell you about $M + 2D + 3T$? How would you interpret this result in terms of the chemical reactions taking place?

6. Show that, in the situation at hand (that is, for the rate equations given at the top of this project), the ratio M/D is *always* decreasing. Hint: use the quotient rule to express $(M/D)'$ in terms of M, D, M', and D'; then use the given rate equations to rewrite your result in terms of M and D only.