TRANSFORMATION MATCHING

Purpose. This is a set of three small group activities, intended to get students thinking about, and discussing, function transformations (shifts, dilations, and reflections), from geometric and algebraic perspectives.

One activity entails transformations of a parabola, another transformations of a trig function, and the third transformations of a logarithmic function.

Preparation (before class) and implementation (in class). There is one activity per page. If two or more of these activities are to be used in the same class on the same day, then each page should be printed on paper of a different color, to help keep the separate activities from getting mixed up.

If possible, pages should be laminated, to protect them so that they may be reused.

Each page (activity) is to be cut along the indicated lines, into a set of fifteen cards. In class, each small group should receive a shuffled set of cards.

Suggested directions. If you wish to make this project as discovery-based as possible, you can distribute the cards (or have them already on the students' tables as they come in) without instructions. Alternatively, you can introduce the project with directions like the following:

> "At your table, you will find a set of fifteen cards. The object of this activity is to group the fifteen cards into five groups of three. Each group of three is to contain a "transformation" card (for example, a card that reads $f(x+1)$), a "formula" card (for example, a card that reads $\cos(2x)$), and a "graph" card (for example, a card that depicts the graph of $y = \ln(x)$). That is: match each of your five graph cards with the formula and the transformation that go with it."

If only a portion of a class period is to be used for group work on this topic, then each group can be given just one activity (one sheet; one type of function) to investigate. If more time is available, have different groups start with different activities, and trade activities with other groups as they complete them.

Leading questions and general ideas. As the students explore this activity, certain questions, like the following, may arise—or you may wish to bring them up to guide the students in their learning.

- What strategies did you use to facilitate the sorting? (For example: students might first notice that there are two graphs that are reflections of each other about the x axis; if they know that this corresponds to replacing x with $-x$, they can fairly easily match two triples, making the rest of the matching easier.)

- Is the "parent" function (the function that is simply denoted $f(x)$, or $g(x)$, or $h(x)$) always the most "basic" function? For example: for the parabolas activity, the parent function is $f(x) = (x-1)^2 = x^2 - 2x + 1$, not x^2. Is $f(x) = x^2$ really any more "basic" than $f(x) = (x-1)^2$?
- (If each group has attempted all three activities.) Rank the activities according to degree of difficulty. What makes a given activity harder, or easier, than the others?
- What are some of the algebra techniques and facts that these activities entail? How is the algebra reflected in the graphs?
- (For the logarithm cards.) Why do all five graphs look the same? What's up with that?

Debrief. If possible, leave some time after the activities are completed for discussion that is more content-focused. This will provide students with the opportunity to understand how the explorations they have just completed apply to the "nuts and bolts" of the topic in question.

Some issues that might be discussed are:

- Given $y = f(x)$, what is the effect, geometrically speaking, of replacing x by $-x$?
- Given $y = f(x)$, what is the effect, geometrically speaking, of replacing x by ax, where a is a positive constant? A negative constant?
- What about replacing x by $x - k$, where k is constant? How does the sign of k figure in?
- Review basic properties of the logarithm function.
- What questions do you have after completing this activity?

As you discuss these questions, you might want to summarize certain facts on the board. For example: make a list of basic properties of the logarithm function. Construct a table of transformations, written algebraically, versus their geometric effects. And so on.