Goal: Use technology to produce a graph of $f(x)=\frac{x^{2}-4}{2 x^{3}+x+1}$, with all key features labeled.

1. Without technology, find the following:

Zeroes of $f(x)$ : $\qquad$
Horizontal asymptote of $f(x)$ : $\qquad$
To find vertical asymptotes, solve the equation:
2. Use technology to find the vertical asymptote(s) of $f(x)$ :
3. Use technology to produce a first graph of $f(x)$. Label the scale on the axes.

4. From the graph, do you think there are local extrema? Explain. Yes/No/Not sure yet
5. From the graph, do you think there are inflection points? Explain. Yes/No/Not sure yet
6. Use technology to calculate $f^{\prime}(x)$. Then use technology to find all critical numbers of $f(x)$.
$f^{\prime}(x)=$
Critical numbers of $f^{\prime}(x)$ :
7. Use technology to graph $f^{\prime}(x)$, then use this graph to classify the critical numbers you found:
$f(x)$ has a local maximum/minimum value of $\qquad$ at $x=$ $\qquad$ because $f^{\prime}(x)$ switches from $\qquad$ to $\qquad$ there.
$f(x)$ has a local maximum/minimum value of $\qquad$ at $x=$ $\qquad$ because $f^{\prime}(x)$ switches from $\qquad$ to $\qquad$ there.
8. From the graph of $f^{\prime}(x)$, how many inflection points do you predict? Explain. none/two/four or more/not sure yet
9. Use technology to calculate $f^{\prime \prime}(x)$. Then use technology to find where $f^{\prime \prime}(x)=0$. Explain why the $x$-values where $f^{\prime \prime}(x)$ does not exist are not important here.
$f^{\prime \prime}(x)=$

Zeroes of $f^{\prime \prime}(x)$ (hint: numerator of $f^{\prime \prime}(x)$ must be zero):

Why zeroes of denominator of $f^{\prime \prime}(x)$ are not important:
10. For each of the zeroes of $f^{\prime \prime}(x)$ you found above, determine whether or not $f(x)$ has an inflection point there. Use the graph of $f^{\prime \prime}(x)$ to justify your conclusions.
11. Return to the graph on the first page and label all key points. Draw a new graph if changing the scale is helpful.

