DERIVATIVE MATCHING

Purpose. This activity is designed to help develop and reinforce students' understanding of the derivative as a function. In particular, the activity addresses the geometric relationships between a function $f(x)$ and its derivative $f'(x)$ (and, sometimes, $f''(x)$).

As described below, this activity is meant to be completed in two stages—the first stage shortly after derivatives have been introduced, and the second during semester review.

Preparation (before class) and implementation (in class). The activity is designed to be done in groups, with three or four students per group.

Before class, copy each page—ideally, each page should be on a different color of paper. The pages can be laminated before cutting up the cards and shuffling.

The first stage of this activity should be done immediately after you've covered derivatives of functions. For this stage, you need only the first two pages—the page with the fifteen graphs labelled $f(x)$, and the page with the fifteen graphs labelled $f'(x)$. (So each student group is to get 30 cards total.) The goal is to pair functions with their derivatives, by considering the slopes of the graphs of $f(x)$ at various points and/or over various intervals.

You might introduce this first stage of the project with directions like the following:

> "The object of this activity is to match the graph of each function $f(x)$ with that of its derivative $f'(x)$. Thus, when you have completed the activity, you will have fifteen matched function/derivative pairs."

(If your cards are color-coded, you can also mention that each pair will comprise cards of two colors.)

Alternatively, if you wish to make this project more discovery-based, you can distribute the activity—or have it waiting on students' tables as they come in—without instructions.

The second stage of this activity should be done as part of semester review. For this stage, use all three pages (45 cards per student group). Each card on the third page contains additional details about one of the fifteen function/derivative pairs. Thus, for this second stage, the goal for each group is to construct fifteen function/derivative/detail triples.

You might introduce this second stage of the project with directions like the following:

> "You may recall that, earlier this semester, you were asked to match graphs of functions $f(x)$ with graphs of their derivatives $f'(x)$. Today's activity asks you match the graph of each function $f(x)$ with that of its derivative $f'(x)$, *and* with a third card describing further details of $f(x)$. Thus, when you have completed the activity, you will have fifteen matched function/derivative/detail triples."

(If your cards are color-coded, you can also mention that each triple will comprise cards of three colors.)

To help manage time for either stage of this activity, it might be helpful to observe that the left-hand column of each page contains the easiest examples, the middle column contains the examples of medium difficulty, and the right-hand column contains the harder examples.

Leading questions and general ideas. As the students explore this activity, certain questions, including but not limited to the following, may arise—or you may wish to bring them up to guide the students in their learning.

- Which pairs (or triples) did you do first, and why? Which did you do last, and why?
- Besides identifying where functions are increasing, decreasing, concave up, and concave down, what else can we use to determine which function has which derivative? (See functions 4 and 7.) How is the *magnitude* of the derivatives relevant?
- What other general strategies or ideas might be helpful in matching a function with its derivative? (For example: consider the domain of $f'(x)$.)
- (For the second stage.) Is it always easiest to first match $f(x)$ with $f'(x)$, and then match a "detail" card to the two, or is it sometimes better to proceed in a different order? Why?

Debrief. If possible, leave some time after the activity is completed for questions, and for discussion of the facts, procedures, and ideas that the activity was meant to reinforce.

Here are some possible takeaways from this activity:

- There are various ways (in terms of the geometric features of $f(x)$) in which the derivative might fail to exist at one or more points. (See functions 3, 8, 10, and 12.)
- Which features of $f'(x)$ correspond to the increase/decrease of f? Which correspond to the concavity (up or down) of f?
- Special things happen to $f(x)$ when $f'(x)$ changes sign, and when $f'(x)$ changes direction (from decreasing to increasing, or vice versa). Don't forget to point out that such things can happen at points x where $f'(x) = 0$ *or* where $f'(x)$ fails to exist.
- What does the derivative of a line look like?
- The degree of a polynomial is related to the degree of its derivative. How? (You might want to help students here by, for example, identifying functions 1, 4, 6, and 7 as first, second, third, and fourth degree polynomials respectively.)
- Which of the fifteen functions are odd; which are even? How does this relate to the parity (evenness/oddness) of the derivative? Challenge question: does $f(x)$ *have* to be even/odd for $f'(x)$ to be odd/even? See functions 1, 2, and 9.
- What shape of curve has a bell curve as its derivative? (The "S" curve depicted here is called a *logistic* curve.)

As you discuss these issues, you might want to write certain facts on the board. For example: make a list of derivative formulas (which can serve as a preview or a summary, depending on whether they've seen such formulas already.) Construct a table comparing properties of f to those of f'. And so on.

Follow-up challenge. Time permitting (or to be completed outside of class), you can ask your students to find possible formulas for each of the fifteen functions.