1. Review and Warm-up: The graph of $f$ is shown below. Calculate exactly each of the definite integrals that follow.

(a) $\int_{0}^{2} f(x) d x=$
(d) $\int_{5}^{8} f(x) d x=$
(b) $\int_{0}^{3} f(x) d x=$
(e) $\int_{2}^{4} f(x) d x=$
(c) $\int_{4}^{5} f(x) d x=$
(f) $\int_{0}^{8} f(x) d x=$
2. Let $s(t)$ be the position, in feet, of a car along a straight east/west highway at time $t$ seconds. Positive values of $s$ indicate eastward displacement of the car from home, and negative values indicate westward displacement. At $t=0$ the car is at home. Let $v(t)$ represent the velocity of this same car, in feet per second, at time $t$ seconds (see graph below).


(a) Write a definite integral representing each of the following:
$s(10)=$
$s(30)=$
$s(t)=$
Now use these integrals and the velocity graph to help you fill in the chart below:

| $t$ | 0 | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $s(t)$ | 0 |  |  |  |  |

(b) Use these values to help you plot the position function.
(c) Fill in the chart below:

| Definite integral of velocity | Change in position |
| :--- | :--- |
| $\int_{0}^{10} v(t) d t=$ | $s(10)-s(0)=$ |
| $\int_{10}^{20} v(t) d t=$ | $s(20)-s(10)=$ |
| $\int_{0}^{40} v(t) d t=$ | $s(40)-s(0)=$ |

(d) Why do these two columns give the same answers?
3. The following data is from the U.S. Bureau of Economic Analysis. It shows the rate of change $r(t)$ (in dollars per month) of per capita personal income, where $t$ is the number of months after January 1, 2012.

| $t$ (months) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $r(t)$ (dollars per month) | 154 | 17 | 10 | 79 | 278 | -432 | 290 |

Use left-hand Riemann sums to estimate the total change in personal income during 2012.
4. A can of soda is put into a refrigerator to cool. The temperature of the soda is given by $F(t)$. The rate at which the temperature of the soda is changing is given by

$$
F^{\prime}(t)=f(t)=-25 e^{-2 t} \text { (in degrees Fahrenheit per hour) }
$$

(a) Find the rate at which the soda is cooling after 0,1 , and 2 hours. Then use this information to estimate the temperature of the soda after 3 hours if the soda was $60^{\circ} \mathrm{F}$ when it was placed in the refrigerator.
(b) Now we will find the exact temperature of the soda after three hours have passed.
i. Find an antiderivative of $f(t)$, that is, find a function $G(t)$ such that $F^{\prime}(t)=f(t)=$ $-25 e^{-2 t}$.
(Hint: take a couple of derivatives of $f(t)$ and try to find a pattern.)
ii. The Fundamental Theorem of Calculus (the Evaluation Theorem) tells us that $\int_{a}^{b} f(t) d t=F(b)-F(a)$. Use this theorem and the function you found in the last step to find the temperature of the soda after 3 hours have passed.
(c) Why do you think your estimate in part (a) is so far off?
5. (a) Write an integral which represents the area between $f(x)=x^{4}$ and the $x$-axis, between $x=0$ and $x=2$.
(b) Evaluate this integral using the Fundamental Theorem of Calculus (the Evaluation Theorem).
6. (a) Using the Fundamental Theorem of Calculus (as in the last problem), evaluate $\int_{0}^{\pi} \cos (x) d x$.
(b) Show the area represented by the integral in part (a) on the graph.

7. Evaluate $\int_{0}^{1} \frac{1}{1+x^{2}} d x$ without using technology.

