

The invariant Hilbert scheme and equivariant degenerations of spherical modules

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Part I: Alexeev and Brion's Invariant Hilbert Scheme Through Examples The invariant Hilbert scheme was introduced by V. Alexeev and M. Brion in 2003 as a new tool for the classification problem of affine algebraic varieties equipped with an action of a complex reductive group G . Given a finite-dimensional G -module V , the invariant Hilbert scheme classifies those closed G -subschemes Z of V whose coordinate ring $O(Z)$ has a prescribed G -module structure. I will introduce this object and some of its basic properties by means of several elementary examples.

Part II: Equivariant degenerations of spherical modules for groups of type A We will work over the complex numbers. When an algebraic group G acts on an affine variety X , the coordinate ring $O(X)$ of X is naturally a G -module. A natural question is whether (or to what extent) the G -module structure of $O(X)$ determines its G -algebra structure. In the setting where G is a reductive linear algebraic group and the G -module $O(X)$ has finite multiplicities, V. Alexeev and M. Brion brought geometry to this question by constructing a moduli scheme which parametrizes the G -multiplication laws "compatible" with the given G -module structure. After briefly reviewing examples of this moduli scheme due to S. Jansou, P. Bravi and S. Cupit-Foutou, I will discuss joint work with S. Papadakis on the case where the given module structure is that of the coordinate ring of a spherical module (i.e. a finite dimensional G -module which is spherical as a G -variety) and G is of type A. In all these examples, the moduli schemes are open subschemes of invariant Hilbert schemes.

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3:00-5:00 p.m.

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