University of Colorado
Boulder

Department of Mathematics
395 UCB
Boulder, Colorado 80309-0395

t 303 492 7664
f 303 492 7707

Kempner Colloquium

Connected components of sphere covers

Michael Fried

(UC Irvine)

By 1872 we knew the space of compact surfaces of genus g was connected. I discuss applications extending this. Consider pairs (X_i, f_i) with f_i, nonconstant and analytic, mapping X_i to the sphere, $i = 1, 2$, both with r > 3 branch points. A locally path connected topology on such pairs uses moving the branch points of the f_is. We ask:

When is (X_1, f_1) connected to (X_2, f_2)?

We find the f_is must have the same geometric monodromy group G with the same r conjugacy classes attached to the branch points. Sometimes these invariants suffice; we get one connected component. Sometimes we need more sophisticated invariants to describe components.
Two problems on rational functions f with coefficients, in a number field K, allow me to show the beginning tools that make this work.

1. Find all (f, K) with the Schur cover property: For infinitely many residue fields of K, $f : w \to f(w) = z$, with w running over the residue field (including ∞), is one-one.

2. Find all (f, K), with f indecomposable over K, but f is a composition of degree > 1 rational functions over the complexes.

We find G is one of two "easy" (centerless) groups: The dihedral group of order $2n$ (n odd) or another slightly larger group, and all $r = 4$ conjugacy classes are the same. Then we see that modular curves include all rational functions giving solutions. Finally, the solutions come from the two fiber types over the j-line in Serre's Open Image Theorem.

This talk is based on §6.1 and §6.2 of "The place of exceptional covers among all diophantine relations," J. Finite Fields 11 (2005) 367–433, arXiv:0910.3331v1 [math.NT]
http://www.math.uci.edu/ mfried/paplist-ff/exceptTower0910-3331v1.pdf

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