Kempner Colloquium

Applications of Cramers formula to probability theory

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The theory of Markov chains on a finite state space is an elaborate story whose main character is a transition probability matrix P, a matrix with non-negative entries each of whose rows sums to 1. Given P, a corresponding Markov chain is a family $\{X_n : n \neq 0\}$ of random variables with the property that, conditional on $(X_0, \ldots, X_n)$, the probability that $X_{n+1} = j$ is $P_{X_n j}$. Thus, at least in theory, every problem that one can pose about the chain can be solved in terms of P once one knows its initial distribution, the distribution of X_0. One such problem is that of finding the stationary distribution π for the chain. That is, the initial distribution with the property that $(X_0, \ldots, X_n, \ldots)$ has the same distribution as $(X_1, \ldots, X_{n+1}, \ldots)$. In this lecture, I will use Cramers formula for determinants to derive an expression for π in terms of the determinants of submatrices of $I - P$. If there is time, I will also discuss how the same ideas lead to an elegant proof of Wilson's algorithm for putting the uniform distribution on the set of spanning trees in a connected graph and, as a corollary, derive Kirchhoff's formula for the number of spanning trees.

Monday, September 30, 2013
4:00 p.m.
MATH 350