MATHEMATICS 2001
GROUPWORK DUE SEPTEMBER 30

Tasks

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

(1) Reminder: You should still probably be able to elect a scribe and presenter who has not yet scribed or presented. If you were elected presenter but did not actually present in class, that doesn't count as having presented, so you may be elected again.

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

   (a) Learn to play the following simple game. Gather some tokens, chips or objects (e.g., coins, candies), and make a pile or grouping of such objects on a table between two players. At each turn, the player takes exactly 1 or 2 tokens from the table (discarding or eating them or whatever), leaving a smaller pile. The player who makes the last move (taking the last token(s) from the pile), wins. Play it for a while.

   (b) Guess a winning strategy and try to write it down explicitly. Test it by playing some more.

   (c) Consider the following definition:

   Definition 1. *A board position (i.e. size of the pile) is called a* winning position *if the first player to move can always win the game, no matter how well his opponent plays. Otherwise it is called a* losing position*.*

   For example, 1 or 2 tokens is a winning position, but 3 tokens is a losing position.

   (d) Gather more data on winning and losing positions, e.g. are 4, 5, 6, etc. winning or losing? Make a table up to 10 or more.

   (e) Determine a correct theorem statement of the following form:

   Theorem 1. *A position of n tokens is a winning position if and only if $n \equiv ? \pmod{?}$.*

   The idea is to put the right things in for the question marks.

   (f) Prove the theorem for $n = 2$, i.e. prove "A position of 2 tokens is a winning position."

   (g) Prove the theorem for $n = 3$, i.e. prove "A position of 3 tokens is a losing position."

   (h) Prove the theorem for $n = 4$.

   (i) Prove the theorem for $n = 5$.

   (j) Prove the theorem for $n = 6$.

   (k) Prove the theorem for $n = 7$.

   (l) Are you getting bored?

   (m) Try to prove the theorem in its entirety (i.e. for all n at once). How could you do this?

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**