MATHEMATICS 2001
GROUPWORK DUE OCTOBER 28

Tasks

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

(1) Reminder: You should elect a scribe and presenter who has scribed or presented the least so far. If you were elected presenter but did not actually present in class, that doesn't count as having presented.

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

(a) This class covers three basic counting principles:

(i) The first is *counting by independent choices*, wherein the counting task can be broken into several independent phases occuring temporally one after the other (e.g. first choose the bread, then the patty . . .). The number of ways to do the entire process is the product of the ways to do each phase.

(ii) The second is *counting by cases*, wherein the counting task can be done in several different ways (cases) (e.g. you can either choose one of four cats, or one of six dogs). The number of ways to do the task is the sum of the ways to do each case.

(iii) The third is *correcting for overcounting*, wherein the counting process can be described by the previous principles, but that description results in overcounting (e.g. you counted ordered pairs but wish to count unordered pairs). The number of ways to do the task is obtained by dividing by the overcounting which has occurred.

For each of these three principles, give an example (of your own creation) of a counting problem which demonstrates it. Think of an audience of your peers, as if you are writing the textbook for this class. How would you explain?

(b) We have encountered the following formula on the inclass worksheets, for the number of ways to choose k objects from n objects:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

You are not allowed in this class, on principle, to know or use any formula without knowing a justification for it. To that end, as a group, please give a proof of this formula. Your proof is an explanation of the formula in terms of a counting process. It can refer to the counting principles above. It can begin something like this: *To choose k objects from n objects, we first choose the first object, which there are n ways to do. . .*

(c) On the last assignment, we saw two different ways to count the number of edges in a complete graph. The first used counting by cases (the answer was a sum), and the second used counting by independent choices, with overcounting (the answer was a product/quotient). Since these two answers are both correct, they must be equal. I've just given you an outline of a *combinatorial proof* of the following theorem:

$$1 + 2 + \cdots + (n-1) = \frac{n(n-1)}{2}.$$

Please write up the proof as a nicely written theorem and proof.

(d) Give a combinatorial proof of the following fact:

$$\binom{n}{k} = \binom{n}{n-k}.$$

Don't use the formula! Give a proof that explains why both sides of this equation count the same thing, hence must be equal. Hint: the left side counts the number of ways to choose k objects from n objects. Why does the right side count the same thing?

(e) Give a combinatorial proof of the following fact:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

(f) Give a combinatorial proof of the following fact:

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}.$$

Hint: you need to figure out the counting problem both sides correctly answer. Feel free to write combinatorial proofs in colorful terms, like talking about committees etc.

(g) Go back and be precise about n and k throughout this assignment. For example, do n and k have to be positive? Make the theorem statements precise.

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**