MATHEMATICS 2001
GROUPWORK DUE OCTOBER 21

Tasks

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

(1) Reminder: You should elect a scribe and presenter who has scribed or presented the least so far. If you were elected presenter but did not actually present in class, that doesn't count as having presented.

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

(a) As always, keep group time to something reasonable (I suggest 3 hours maximum), and if you don't finish everything in that time frame, that is ok.

(b) Read pages 163-164 of Hammack, about graphs. Read together, and then explain to each other the key idea of the proof by talking and drawing diagrams, until everyone understands.

(c) A *complete* graph on n vertices is the graph with n vertices and an edge between every single pair of vertices. In other words, it has all possible edges. Prove by induction that the number of edges in a complete graph on n vertices is $n(n-1)/2$.

(d) Now prove the same theorem using a counting argument. That is, give a counting procedure for the edges which results in that formula (like on our counting problem worksheets). Write it up as a proof.

(e) Counting question: how many different graphs are possible on n vertices? (Imagine the vertices are labelled 1 through n, so they are distinguished.) Can you give a proof?

(f) The *degree* of a vertex is the number of edges coming out of it. Prove that the sum of the degrees of the vertices of any finite graph is even.

(g) A graph is *connected* if you can get between any two vertices by travelling along edges. Fact: Any connected graph with at least two vertices has some vertex, which, when removed, leaves the graph connected. Prove this, and, as always, write it nicely. (When a vertex is removed, the edges connected to it also vanish.)

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**