

MATHEMATICS 2001
GROUPWORK DUE SEPTEMBER 30

TASKS

Reminder: you should produce a Groupwork Report (handwritten is fine) and a PDF uploaded to D2L (typset, LaTeX or Word or whatever).

Reminder: elect a leader, scribe and presenter.

- (1) Reminder: You should elect a scribe and presenter who has scribed or presented the least so far. If you were elected presenter but did not actually present in class, that doesn't count as having presented.
- (2) **Main Task 1: Take up homework done so far.** As in previous weeks.
- (3) **Main Task 2: Group Homework.**
 - (a) At last, a really non-obvious theorem to prove! Consider the sum

$$S_k = \sum_{k=1}^n 1/k.$$

For example, $S_1 = 1$, $S_2 = 1 + 1/2 = 3/2$. Prove that this sum can never again be an integer, for any $k > 1$. *Hint:* Use contradiction. Try multiplying up by *almost* the largest power of two among the denominators, and think about even and odd denominators. Try an example like multiplying S_5 by 2 (since 4 is the largest power of 2 less than or equal to 5).

- (b) Prove that $5^n - 1$ is always a multiple of 4. The proof is by induction. Hint: $5^n - 1 = 4 \cdot 5^{n-1} + (5^{n-1} - 1)$.
- (c) Play the lovely game of CHOMP! available here:
<https://www.math.ucla.edu/~tom/Games/chomp.html>

Play against each other a little (using paper or tokens or something), with different size starting chocolate bars.

- (d) Prove by contradiction that any rectangular chocolate bar bigger than 1×1 is a winning position. Hint: The proof proceeds by 'strategy stealing' and gives you no idea what the winning strategy actually is! If the first player doesn't have a winning strategy, then she can eat one little corner of the bar and watch what her opponent, who does have a winning strategy, does next.
 - (e) Can you figure out the winning strategy for square boards?
- (4) Fill out your groupwork report and have everyone sign. **This is due in class.**
 - (5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**