MATHEMATICS 2001
GROUPWORK DUE NOVEMBER 4

Tasks

(1) If you were elected presenter but did not actually present in class, that doesn't count as having presented. Check D2L. If you have anyone in your group who needs to present, they can email me to make sure it happens.

(2) **Main Task 1: Take up homework done so far.** As in previous weeks.

(3) **Main Task 2: Group Homework.**

   (a) On Friday, one group introduced us to Inclusion-Exclusion (i.e. additive overcounting). Read Section 3.5 of your text as a group, up to the end of Example 3.8. Discuss this example. The question given in class was: how many integers between 1 and 200 are multiples of 3 or 4? Write up an answer to this problem in the language of $|A \cup B|$, $|A \cap B|$, $|A|$ and $|B|$.

   (b) How many four-letter words using english letters (nonsense words allowed) either don't begin with A or don't end with Z? Give a justification. (Hint: Inclusion-Exclusion AKA additive overcounting like the last problem.)

   (c) If you gave an algebra proof of the following fact on the last groupwork, now **give a combinatorial** proof of the following fact:

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}.$$

   Hint: you need to figure out the counting problem both sides correctly answer. Feel free to write combinatorial proofs in colorful terms, like talking about committees etc. Also, I've put up an example combinatorial proof solution online on the website, so you have some examples to look at. If you did this on the last assignment, you can just re-use that answer to hand in, or you can streamline it and make it better. Also, I've noticed groups don't read the instructions very carefully (I'm looking at all of you who didn't do combinatorial proofs last time), so here's an easter egg for those that do: if you can work a Star Trek reference into your proof, I'll give you a bonus…something. No guarantee it's a point.

   (d) Review Section 7.3 of the text. Prove that there exists an integer m so that $m+8 = 5m$. (Hint: this is easy.)

   (e) Prove that every odd integer is the sum of two consecutive integers. (Hint: this is an existence proof.)

   (f) Using the last question if you want, prove that *an integer is odd if and only if it is the sum of two consecutive integers.* (Hint: review Section 7.1 of the text).

   (g) An integer x is called an *additive identity* if it has the property that $x + y = y$ for all integers y. You can think of one such integer, namely zero. Prove that the integers have a unique additive identity. (Hint: Review Section 7.3 of the book. First simply note that 0 is an additive identity (existence). Then, to show uniqueness, suppose there are two additive identities, m_1 and m_2, and show that $m_1 = m_2$ (using the definition of additive identity).)

(4) Fill out your groupwork report and have everyone sign. **This is due in class.**

(5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**