

MATHEMATICS 2001
GROUPWORK DUE NOVEMBER 18

TASKS

- (1) If you were elected presenter but did not actually present in class, that doesn't count as having presented. Check D2L. If you have anyone in your group who needs to present, they can email me to make sure it happens.
- (2) **Main Task 1: Take up homework done so far.** As in previous weeks.
- (3) **Main Task 2: Group Homework.**
 - (a) Study this Definition: Two sets are of the *same cardinality* (in other words, the same size), if and only if there is a bijection (bijective function) between them. If A and B are of the same cardinality, then we write $|A| = |B|$. For example, $A = \{1, 2\}$ and $B = \{a, b\}$ are the same size because there is a bijection $f : A \rightarrow B$ given by $f(1) = a$, $f(2) = b$. There are other bijections what would accomplish the same thing.
 - (b) Prove formally that *have the same cardinality* is an equivalence relation.
 - (c) What is the equivalence class of the set $\{1, 2, 3, 4\}$? (I'm looking for an informal answer.)
 - (d) Prove that the set of even integers and the set of odd integers are of the same cardinality. In other words, write down a function between the two sets, and prove it is a bijection.
 - (e) Prove that the set of even integers and the set of integers are of the same cardinality.
 - (f) Prove that the set of positive integers and the set of integers are of the same cardinality.
 - (g) Here are some sets. Do your best to place them into equivalence classes according to cardinality (i.e. figure out which are the same size). Hint: they are NOT all the same size.
 - (i) \mathbb{N} (positive integers)
 - (ii) \mathbb{Z}
 - (iii) the prime numbers
 - (iv) $\mathbb{Z} \times \mathbb{Z}$
 - (v) $\mathbb{Q} \cap [0, 1]$ (the rationals in the unit interval)
 - (vi) \mathbb{Q}
 - (vii) \mathbb{R}
 - (viii) functions $f : \mathbb{N} \rightarrow \mathbb{N}$
 - (h) Provide as much proof as you can of your conjectured equivalence classes. Write down bijections, or prove they don't exist.
- (4) Fill out your groupwork report and have everyone sign. **This is due in class.**
- (5) The scribe will prepare a PDF of your proofs to hand in on D2L. **I appreciate getting these early on Friday so I can look through them.**